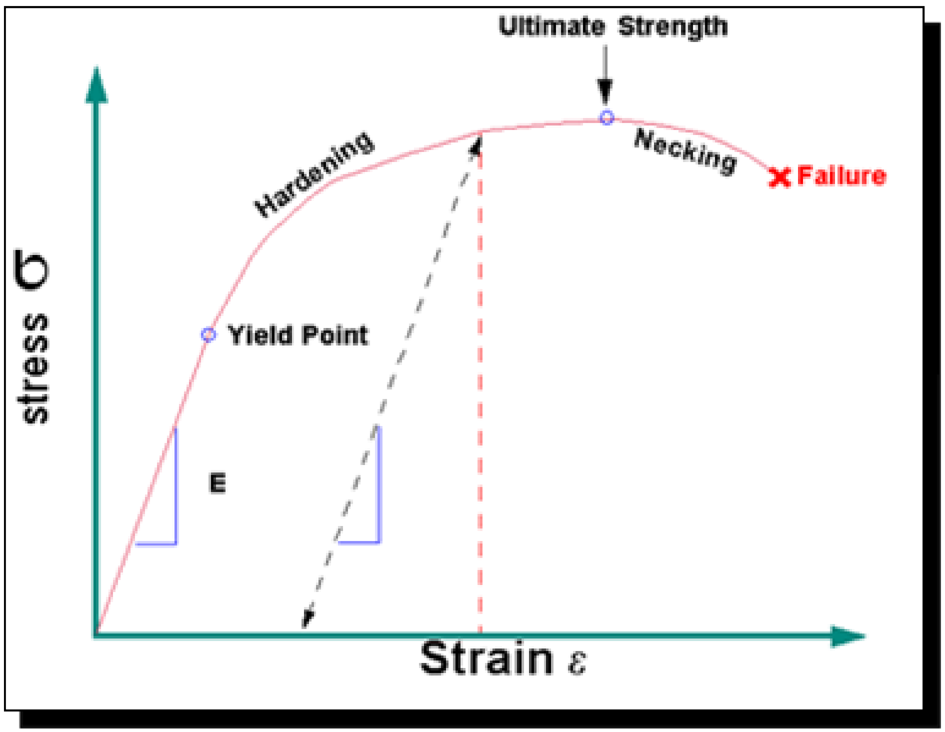
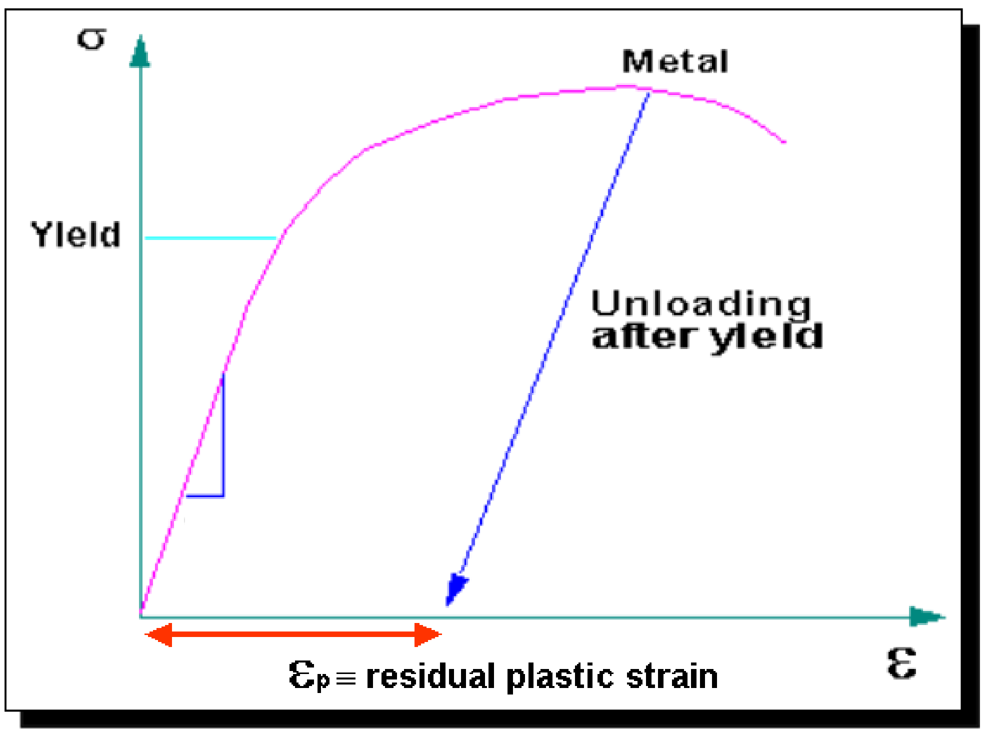
Plasticity

- As stresses (and strains) are increased beyond the yield point, metals start to exhibit a nonlinear behavior: Plasticity
- The yield point defines the shift from elastic to plastic behavior for a material



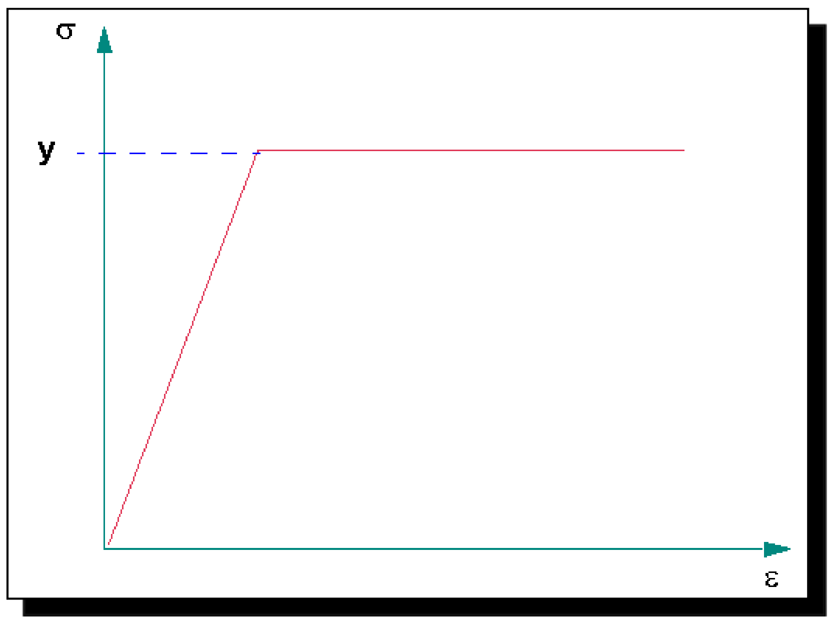
Plasticity

- Loading beyond the yield stress induces permanent (plastic) deformation
  - A yielded ductile metal will unload along a curve that is parallel to the initial linear elastic curve
  - For metals, the yield stress usually occurs at .05% - .1% of the material's Elastic Modulus



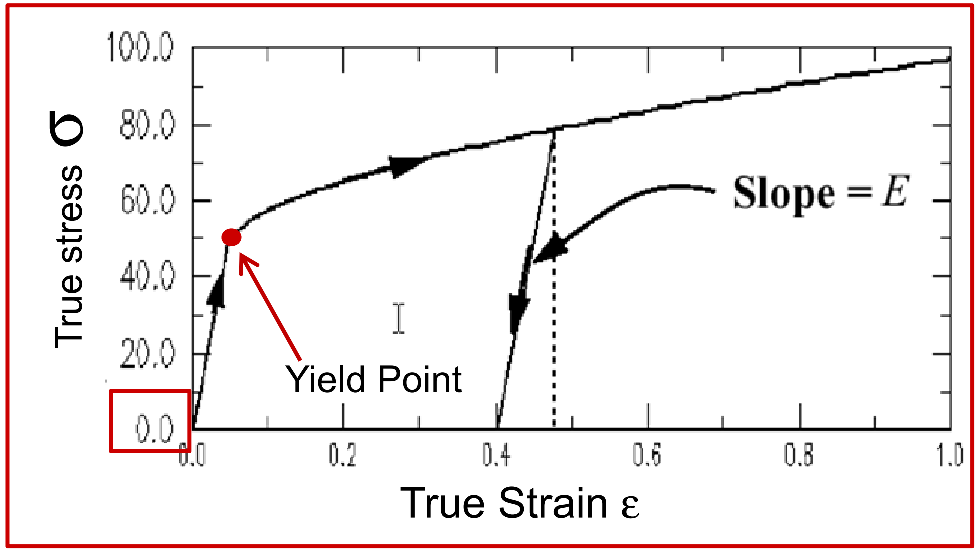
Elastic – Perfectly Plastic Material

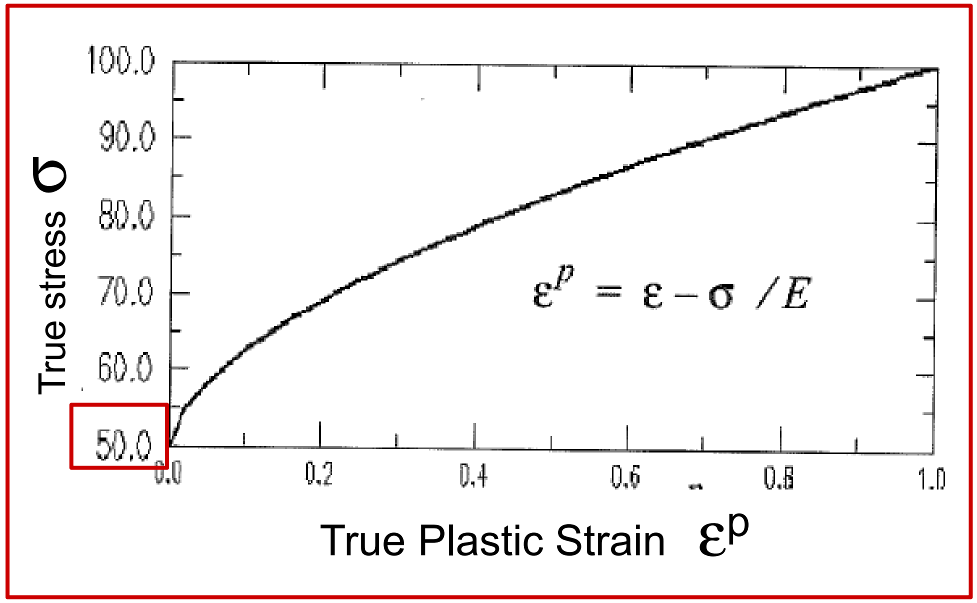
- Simplest form of elastic-plastic behavior is elastic –perfectly plastic behavior
- Assumes the stresses above yield are constant, as shown in diagram below
- Perfectly plastic is good for modeling first order material plasticity
- The yield value may be a function of temperature



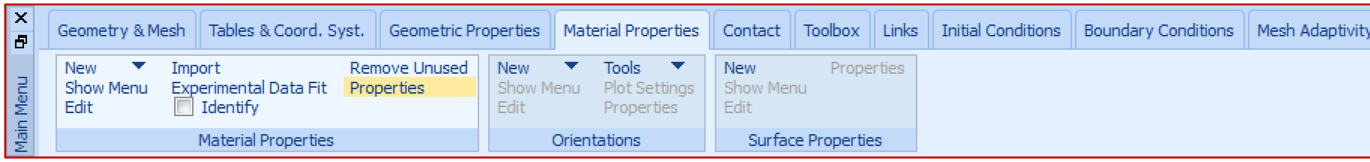
General Plasticity

- Marc expects data for the elastic and plastic range to be entered separately.
- Plastic data should be entered in terms of true stresses and true plastic strains rather than in terms of engineering stresses and engineering strains. (Discussed in detail later)
- The plastic portion requires selecting a Hardening Law.

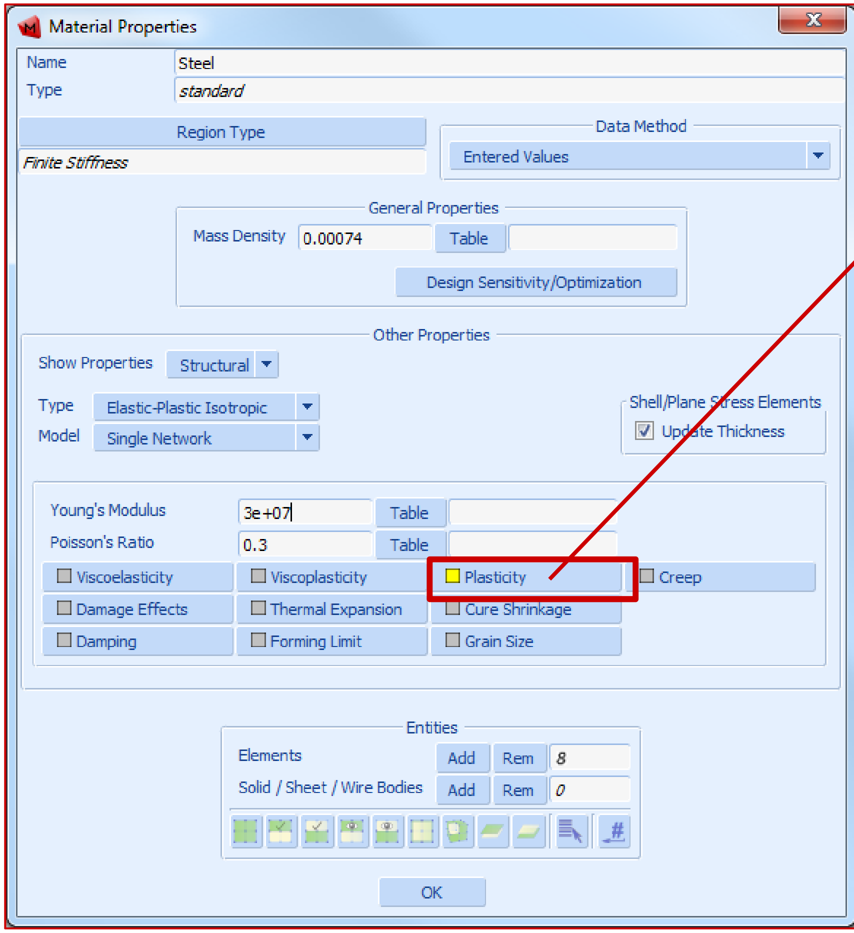


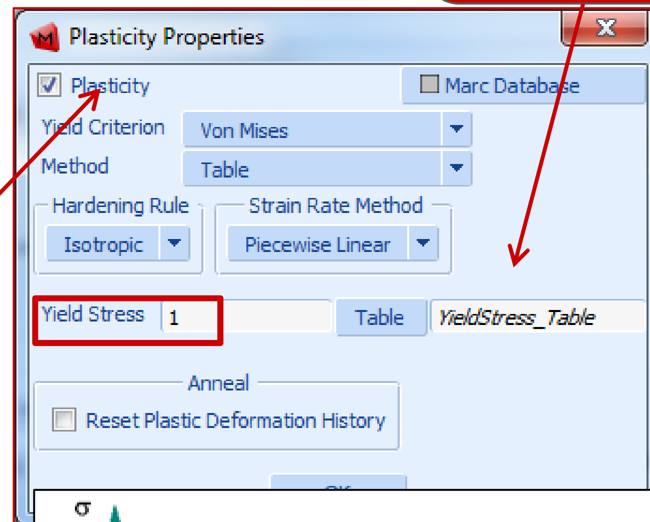


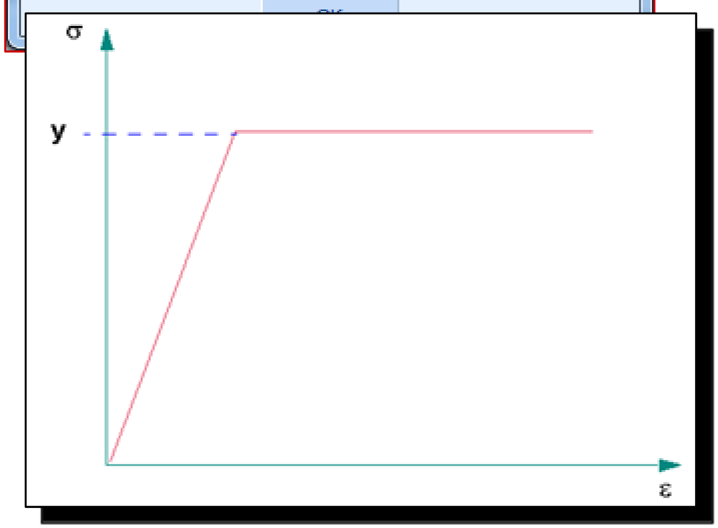
Defining Plasticity of a Material in Mentat



Note the empty field here for elastic-perfectly plastic option





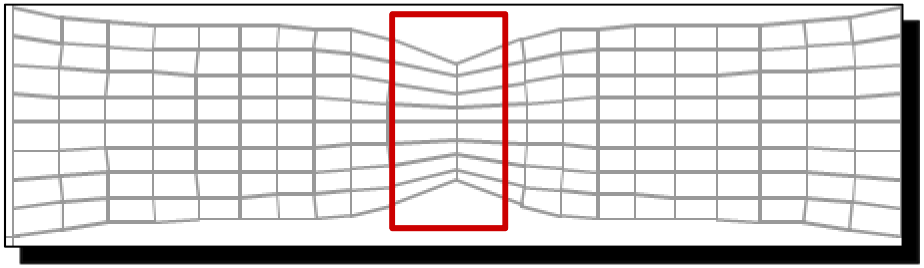


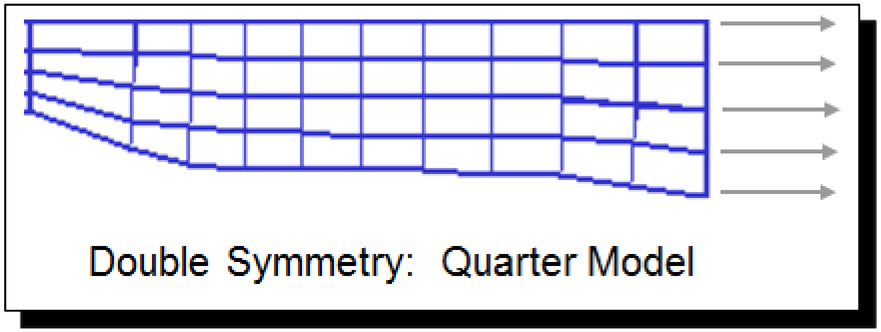
Measures of Stress and Strain

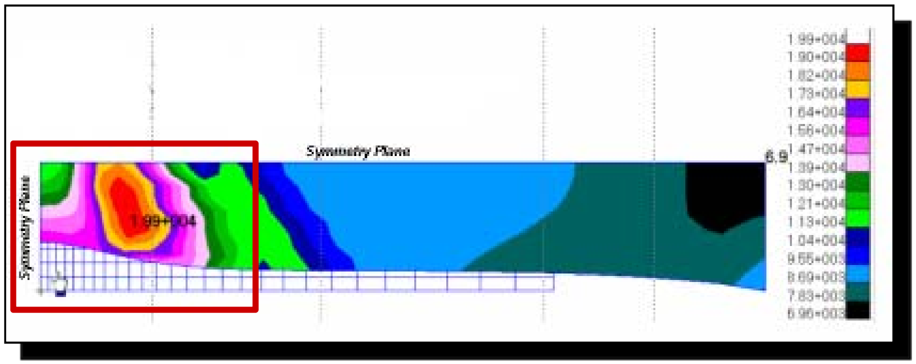
- As noted previously Marc expects data in the form of true stresses vs. true plastic strains
- It is imperative the analyst recognize that there is a difference between stress and strain measures
- Specifically the difference between engineering strain/stress and true strain/stress is highly important
- The differences are best observed by reviewing the phenomena of ‘Necking’ - describe on the following page

Necking

- At high strains, a metal may experience highly localized extension and thinning, usually called necking.
- The nominal stress of a metal as it is necking is much lower than its ultimate strength.
- This behavior is due to the following factors:
  - The geometry of the specimen
  - The nature of the test itself (that is tension or compression)
  - The stress and strain measure used (that is nominal stress and strain)
- This behavior is best represented by Isotropic Hardening







Engineering Stress and Strain

- Engineering stress and strain are computed using the un-deformed shape dimensions as reference values
- Engineering (nominal) stress:

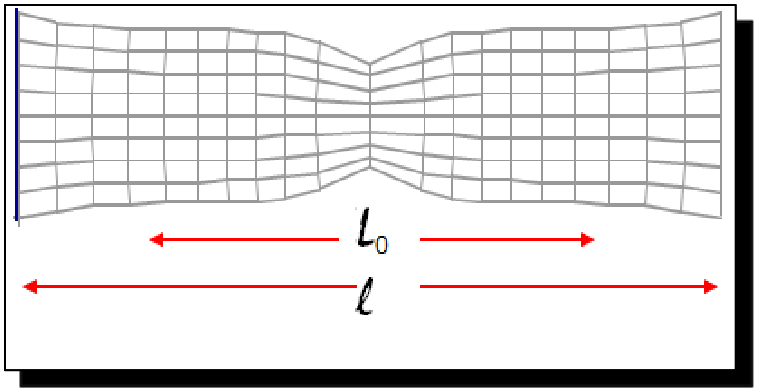
$$\sigma_{nom} = \frac{F}{A_0}$$

  – Where: σ_{nom} is the engineering stress
  F is the force
  A_0 is the undeformed or initial area

- Engineering (nominal) strain:

$$\varepsilon_{nom} = (l - l_0)/l_0$$

  – Where:
  ℓ_0 is the initial (undeformed) length
  ℓ is the length at the time the strain is measured



- Typically, laboratory measurements are expressed in engineering stresses and strains

True Stress and True (Log) Strain

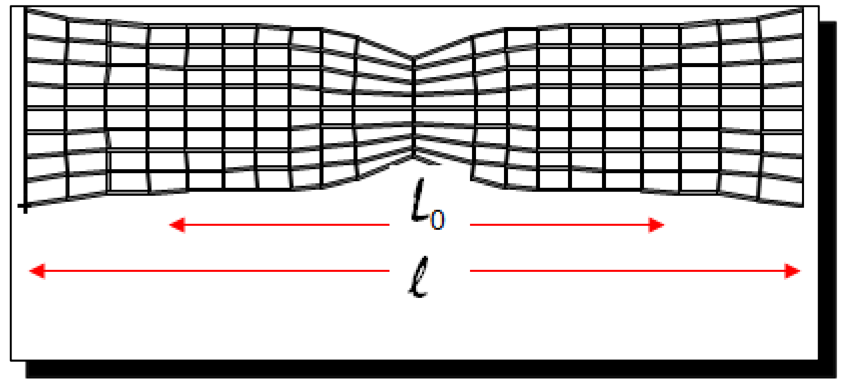
- True stress and true strain are defined as follows:
- True stress:

$$\sigma_{true} = \frac{F}{A}$$

  – A is the "instantaneous area"
- True strain:

$$\varepsilon_{true} = \int_{l_o}^{l} \frac{dl}{l} = \ln \frac{l}{l_o}$$

  – ℓ_o is the initial length
  – ℓ is the "instantaneous length"
- Plastic laws must be entered as true stress and true strain values in the table forms.



- Marc uses true stress and true strain to account for changes in area during finite deformations, which results in a more accurate mathematical model.

Plastic Stress and Strain Data

- Plasticity definition in Marc is defined as the post-yield, or plastic portion of the stress-strain curve
- Typical engineering data for stress-strain curves are defined as total nominal strain

$$\sigma_{true} = \sigma_{nom}(1+\varepsilon_{nom})$$
$$\varepsilon_{true} = \ln(1+\varepsilon_{nom})$$

- The first point of a plastic strain definition represents the yield point, corresponding to a plastic strain value of zero. Thus, when creating a material property table in Mentat, subtract the elastic strain at each point from the total true strain values:

$$\varepsilon_{total} = \varepsilon_{elastic} + \varepsilon_{plastic}$$

or alternatively stated

$$\varepsilon_{plastic} = \varepsilon_{total} - \varepsilon_{elastic}$$

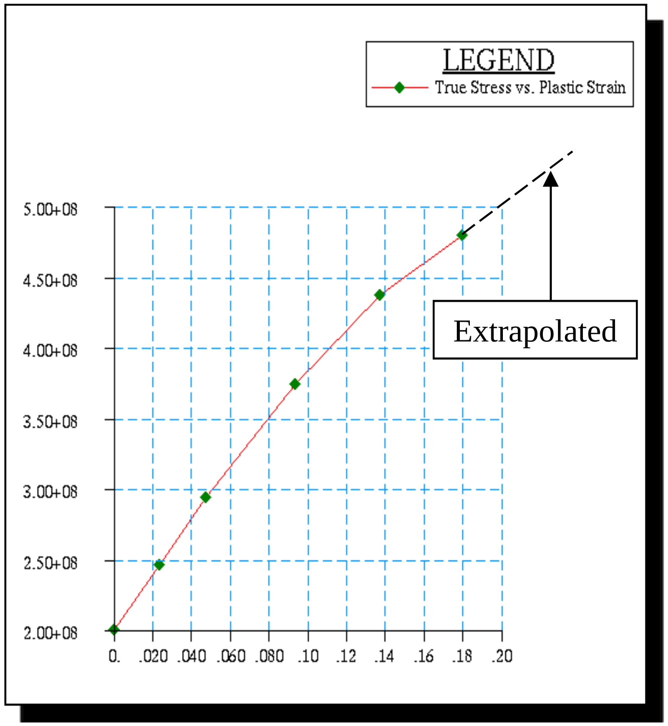
  – Thus:

$$\boxed{\varepsilon_{plastic} = \varepsilon_{total} - \sigma_{true}/E}$$

Converting Engineering Material Data

- Example of converting material test data to valid Marc inputs

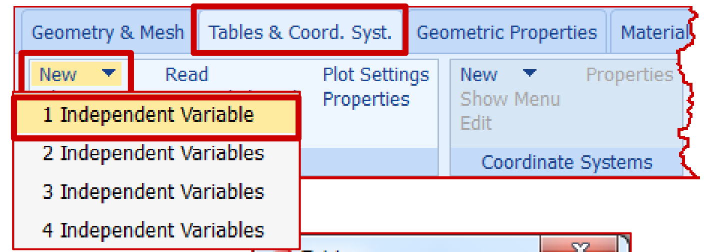
Nominal Stress σ_{nom}	Nominal Strain ε_{nom}	True Stress $\sigma_{nom}(1+\varepsilon_{nom})$	Total True Strain $\ln(1+\varepsilon_{nom})$	Plastic Strain $\varepsilon_{tot}-\sigma_{true}/E$
200E6	0.00095	200.2E6	0.00095	0.0000
240E6	0.0250	246.0E6	0.0247	0.0235
280E6	0.0500	294.0E6	0.0488	0.0474
340E6	0.1000	374.0E6	0.0953	0.0935
380E6	0.1500	437.0E6	0.1398	0.1377
400E6	0.2000	480.0E6	0.1823	0.1800

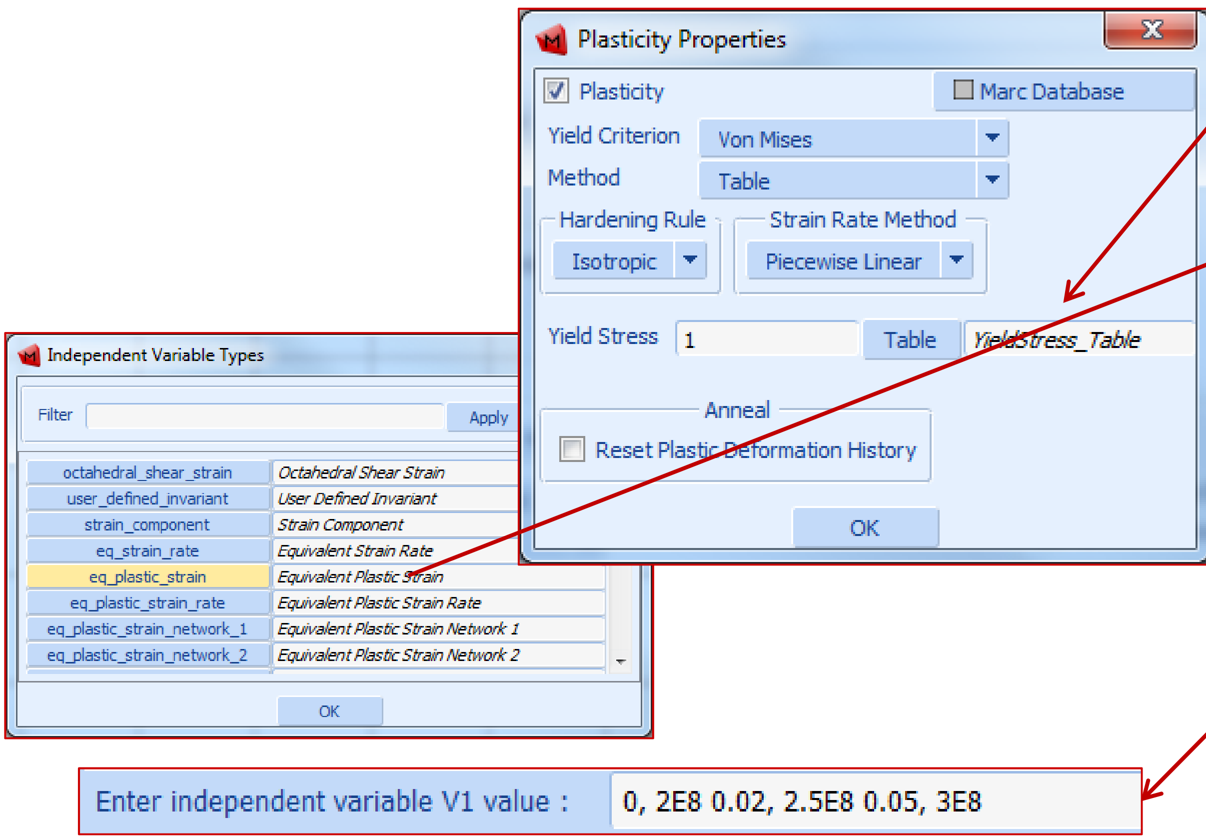


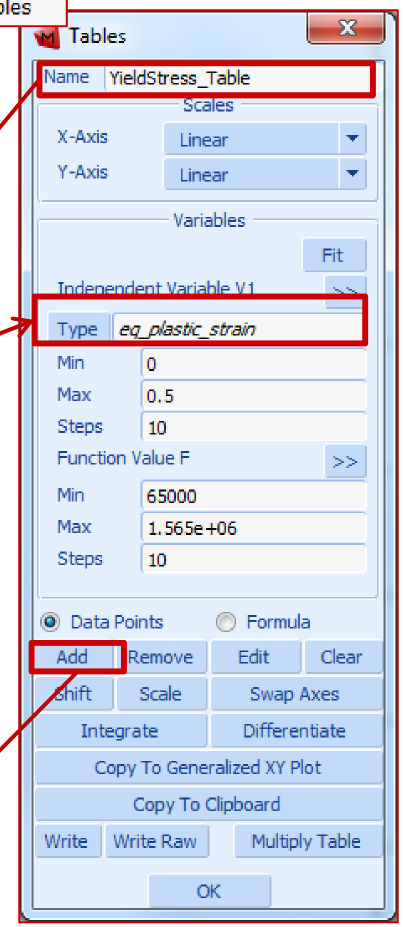
- Linear interpolation is used between data points
- The material's response is extrapolated outside the specified range

Plastic Stress and Strain Data

- Defining plasticity data within Mentat
  - Make all appropriate stress/strain conversions
  - Create a table within Mentat that defines true plastic strain vs. true stress

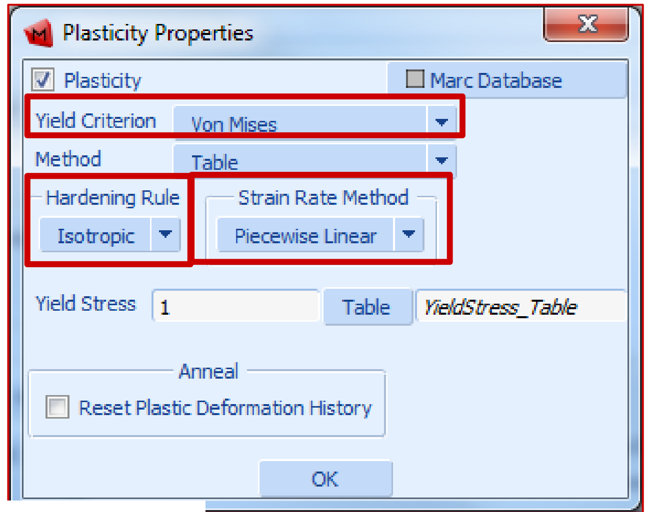


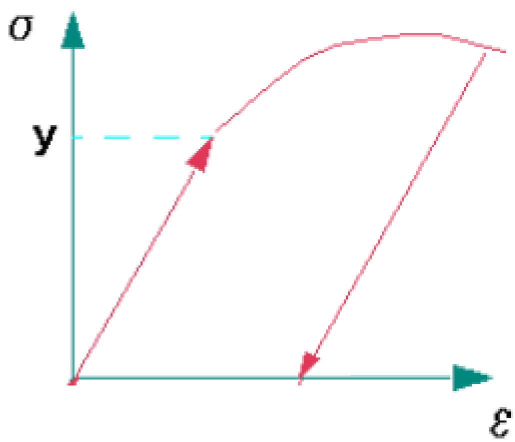




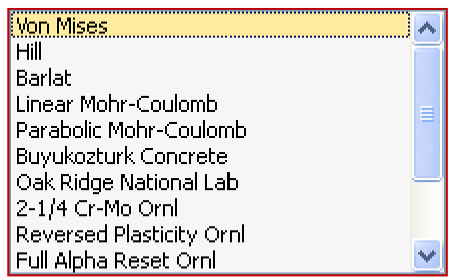
General Elastic-plastic Material

- In addition to yield stress and plastic strain vs. true stress curve the user must specify:
  - Yield Criterion
    - Default – Von Mises
  - Hardening Laws
    - Default - Isotropic
  - Strain Rate Method
    - Default - No strain rate dependence

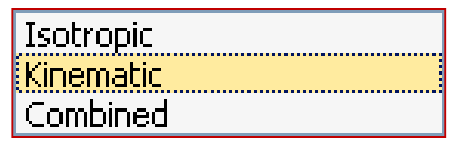




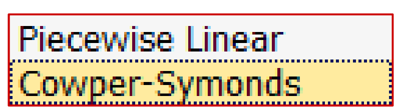
Yield Criterion

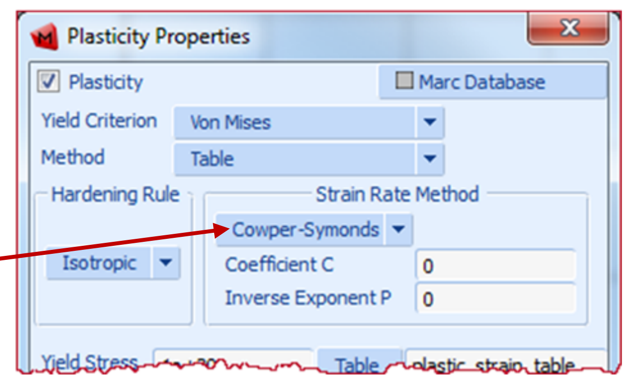


Hardening Rules



Strain Rate Method





Hardening Laws

- Hardening laws define how the yield point changes after initial yielding AND a reversal of loading
- Ideally Plastic:

$$F(\sigma_{ij}) - \sigma_y = 0$$

- Isotropic Hardening:

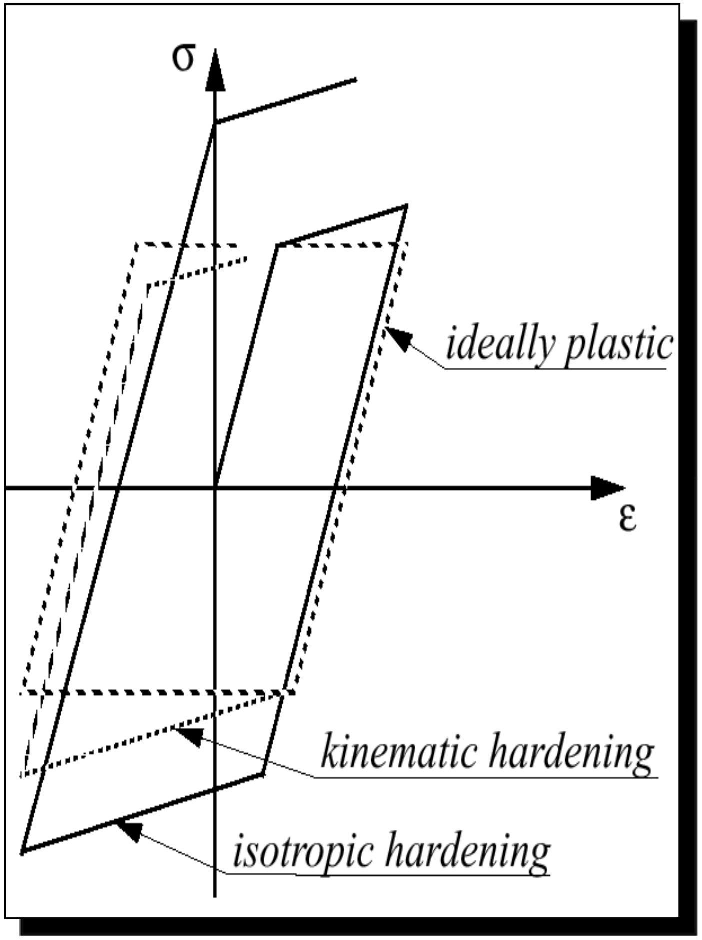
$$F(\sigma_{ij}) - \sigma_y(\bar{\varepsilon}^p) = 0$$

- Kinematic Hardening:

$$F(\sigma_{ij} - \alpha_y) = 0$$

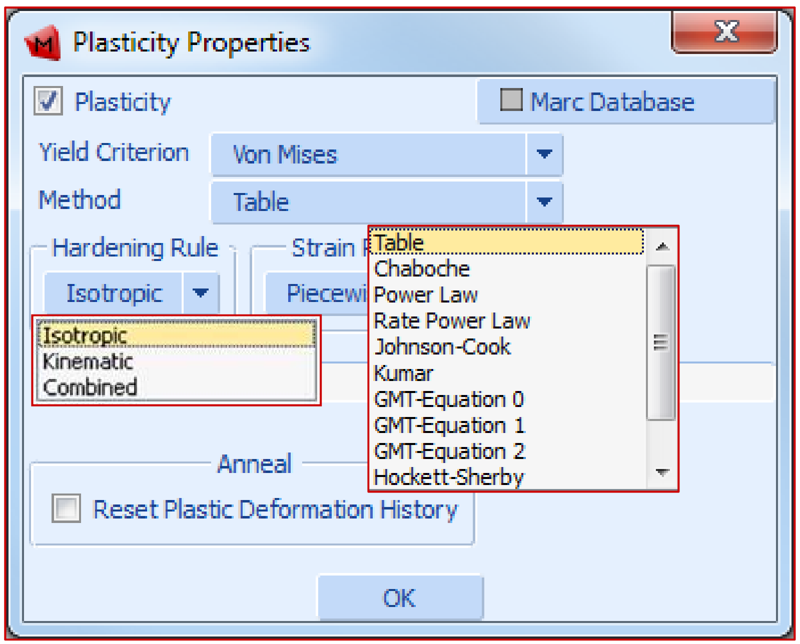
- Combined:
  – Starts as Isotropic and continues as Kinematic

$$F(\sigma_{ij} - \alpha_{ij}) - \sigma_y(\bar{\varepsilon}^p) = 0$$

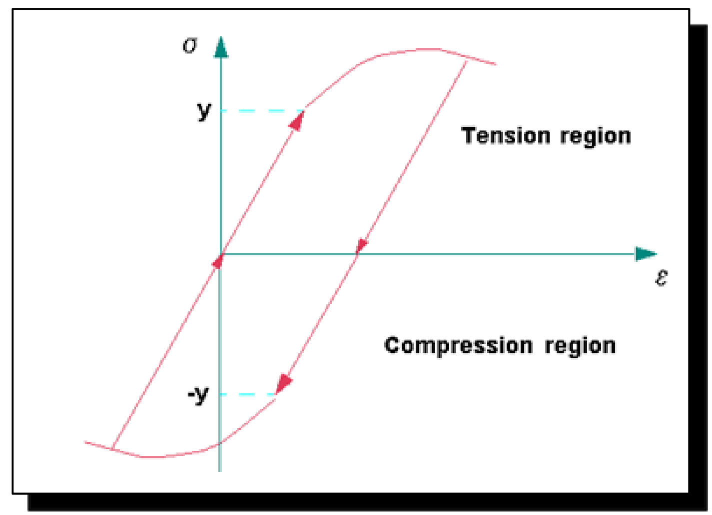


Isotropic Hardening

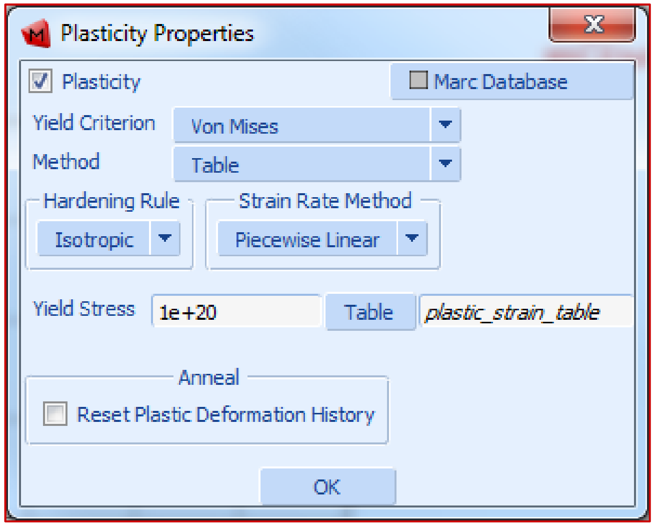
- Isotropic Hardening is good for modeling plasticity, where material flow is the predominant effect that is being captured.
- No shift in stresses - cannot be used to model hysteresis.
- Commonly used to model drawing or other metal forming operations.



Hardening Rule on the Plasticity Properties Form



Von Mises Yield and Isotropic Hardening



- For an isotropic material

$$\bar{\sigma}=[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]^{1/2}/\sqrt{2}$$

- where σ_1, σ_2 and, σ_3 are the principal Cauchy stresses. It can also be expressed in terms of non-principal Cauchy stresses as follows:

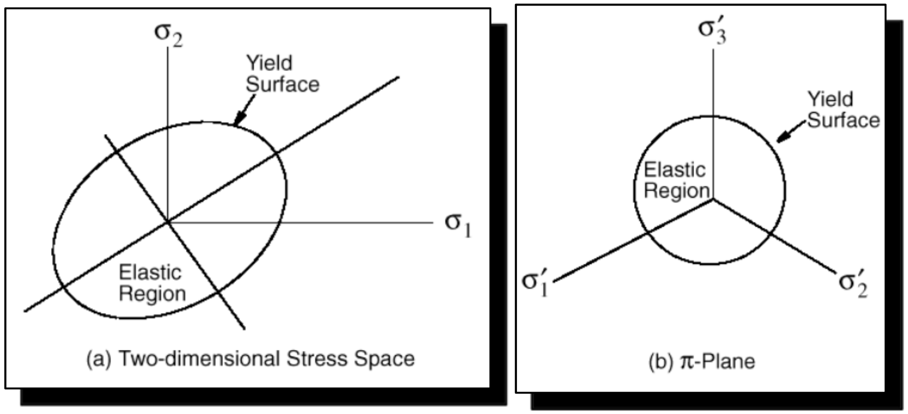
$$\bar{\sigma}=[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)]^{1/2})/\sqrt{2}$$

- The yield condition can also be expressed in terms of the deviatoric stresses as:

$$\bar{\sigma}=\sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$$

- Where σ'_{ij} is the deviatoric Cauchy stress expressed as

$$\sigma'_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij}$$



Von Mises Yield and Isotropic Hardening

- Isotropic Work Hardening assumes that the center of the yield surface, Figure a, remains stationary in the stress space, but that the size (radius) of the yield surface expands due to work hardening.
- Figure b depicts the load path of a uniaxial test with loading and unloading of a specimen
  - It is first loaded from stress free, point 0, to initial yield at point 1
  - it is then continuously loaded to point 2
  - Then unloading from 2 to 3 following the elastic slope E (Young’s modulus)
  - Elastic reloading from 3 to 2 takes place
  - The specimen is plastically loaded again from 2 to 4.
  - Elastically unloaded from 4 to 5
  - Reverse plastic loading occurs between 5 and 6
- Other comments for Figure b
  - Stress at 1 is the yield stress and stresses at points 2 and 4 are larger than yield due to work hardening
  - During unloading, the stress state can remain elastic as in point 3, or it can reach a subsequent (reversed) yield point (for example, point 5)
  - The isotropic work hardening rule states that the reverse yield occurs at current stress level in the reversed direction. Take the stress level at point 4; then the reverse yield takes place at point 5 (negative of the stress at Point 4)

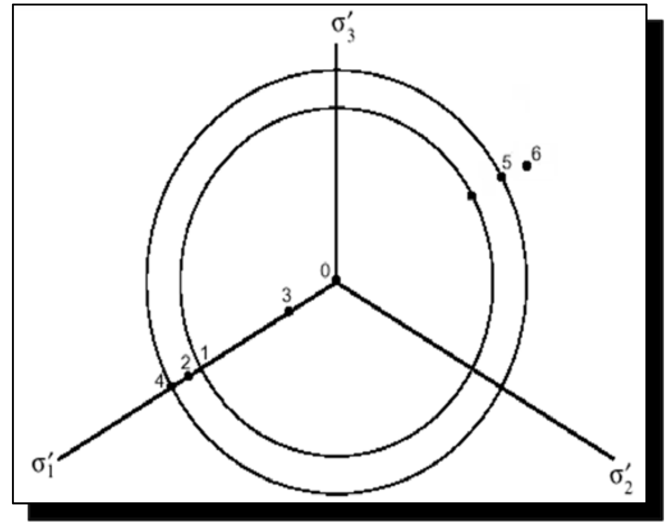


Figure a: Von Mises Yield Surface

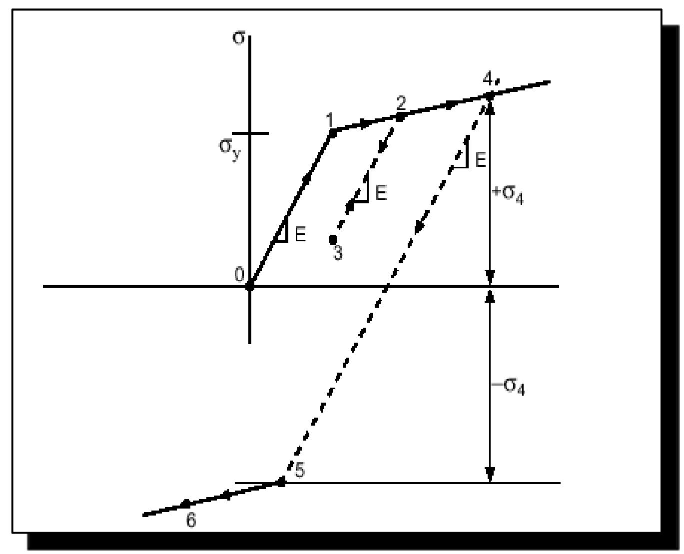
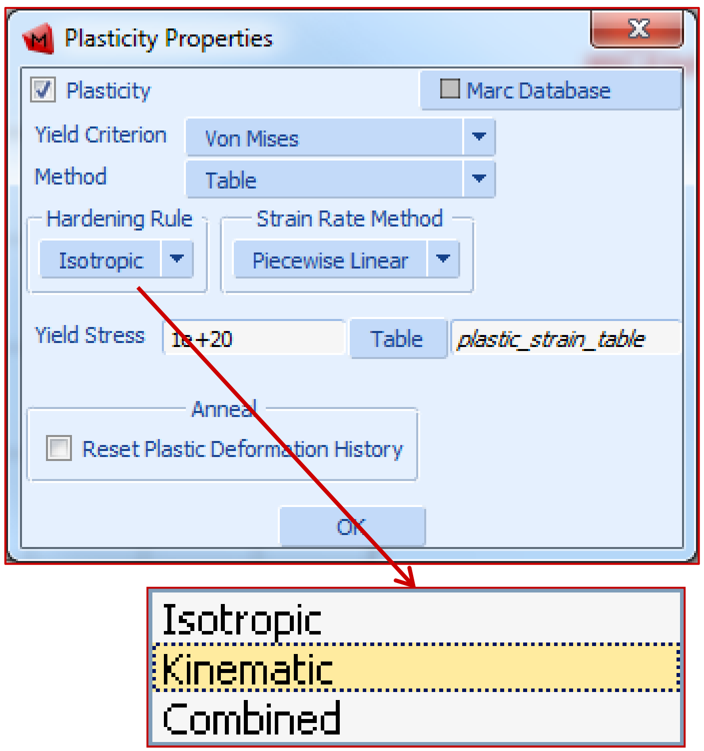
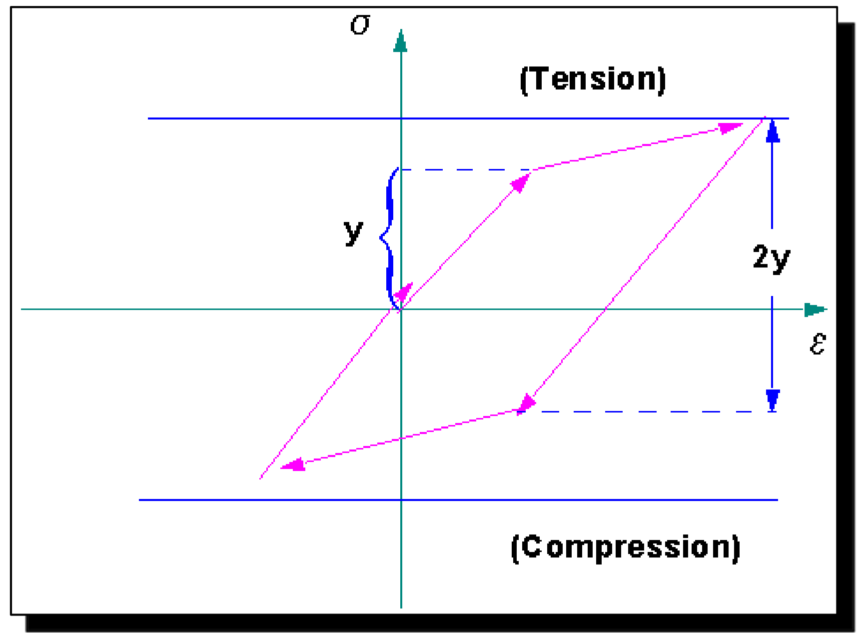


Figure b: Loading Path

Kinematic Hardening

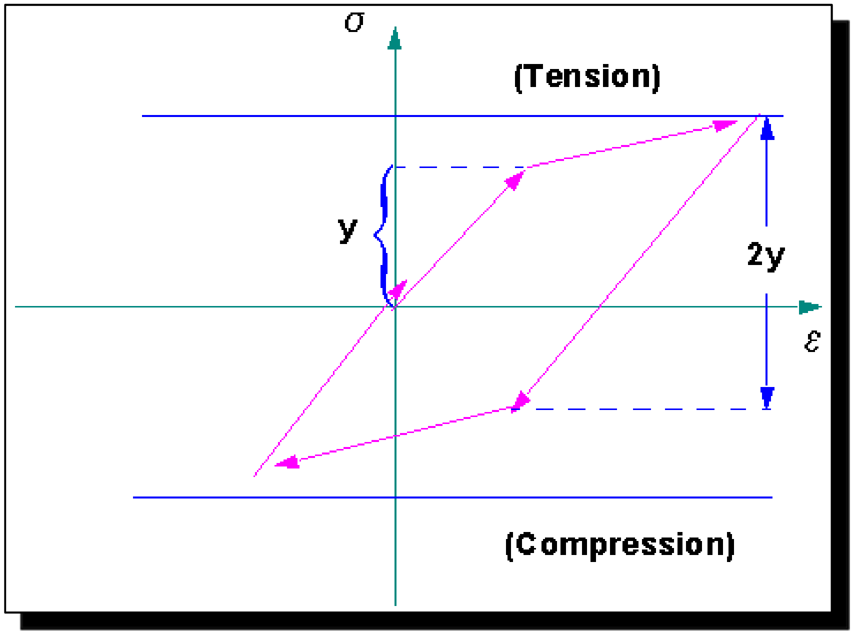
- Kinematic Hardening is good for simulating loading and unloading effects where the compression yield is less than the tension yield due to hardening, as depicted in the figure below.
- The plastic deformation of a material will often increase its yield stress for subsequent loadings.





Kinematic Hardening

- For many materials, the Kinematic Hardening model gives a better representation of loading/unloading behavior than the isotropic hardening model. For cyclic loading, however, the kinematic hardening model cannot represent either cyclic hardening or cyclic softening.
- Kinematic Hardening is not applicable for metal forming simulations in which there is significant plastic flow and straining is generally monotonic.



Von Mises Yield and Kinematic Hardening

- Figure a illustrates the Kinematic Hardening rule. The von Mises yield surface does not change in size or shape, but the center of the yield surface can move in stress space.
- The loading path of a uniaxial test is shown in Figure b. The specimen is loaded in the following order
  - From stress free, point 0, to initial yield at 1
  - It is then continuously loaded to 2
  - Unloading is from 2 to 3
  - Elastic reloading takes place from 3 to 2
  - Plastic reloading takes place from 2 to 4
  - Elastic unloading occurs from 4 to 5
  - Reverse plastic occurs from 5 to 6.
- Other comments for Figure b
  - As in isotropic hardening, stress at 1 is equal to the initial yield stress, and stresses at 2 and 4 are higher than the initial yield stress, due to work hardening.
  - Point 3 is elastic, and reverse yield takes place at point 5
  - Under the kinematic hardening rule, the reverse yield occurs at the level of $\sigma_5 = (\sigma_4 - 2\sigma_y)$, rather than at the stress level of $-\sigma_4$
  - Similarly, if the specimen is loaded to a higher stress level, point 7, and then unloaded to the subsequent yield point 8, the stress at point 8 is $\sigma_8 = (\sigma_7 - 2\sigma_y)$
  - If the specimen is unloaded from a tensile stress state (such as point 4 and 7), the reverse yield can occur at a stress state in either the reverse direction (point 5) or the same direction (point 8).

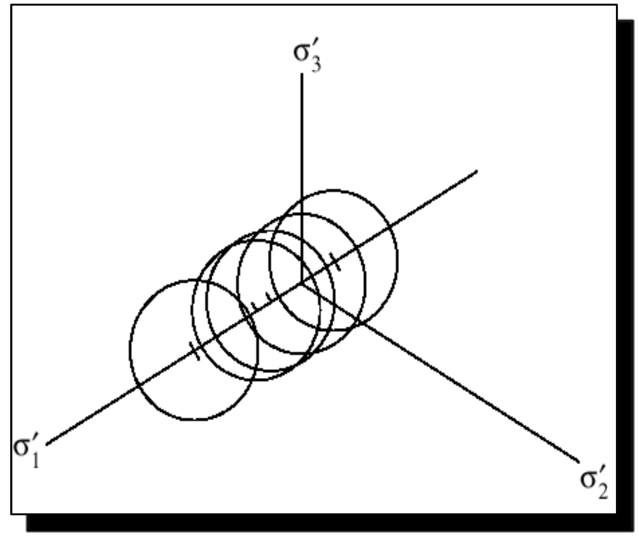


Figure a: Von Mises Yield Surface

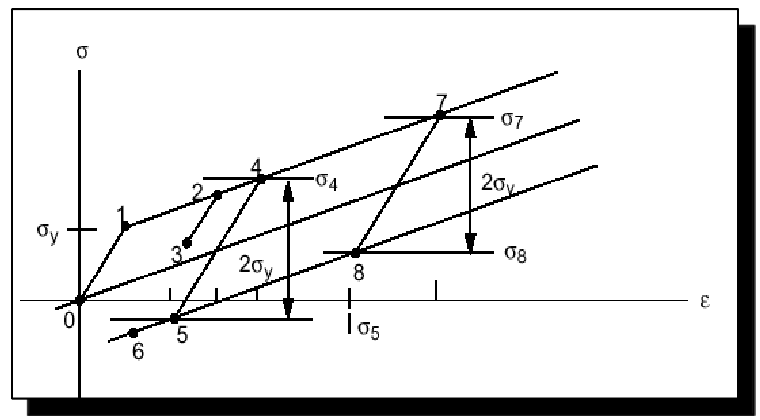
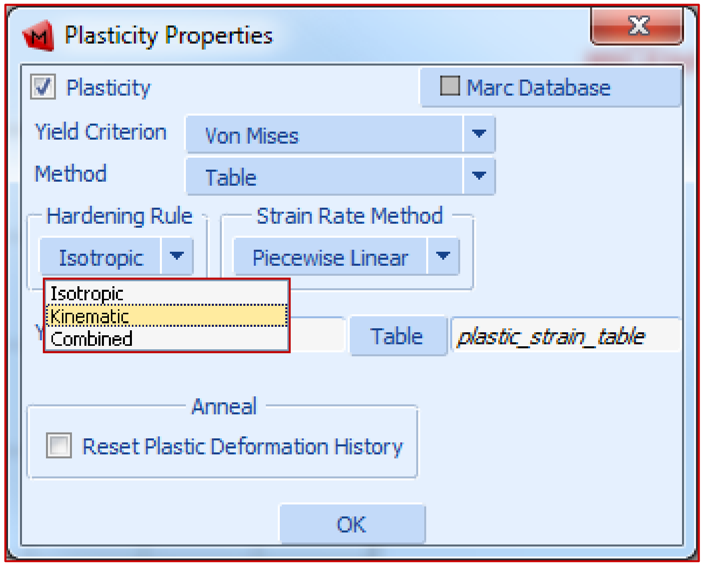
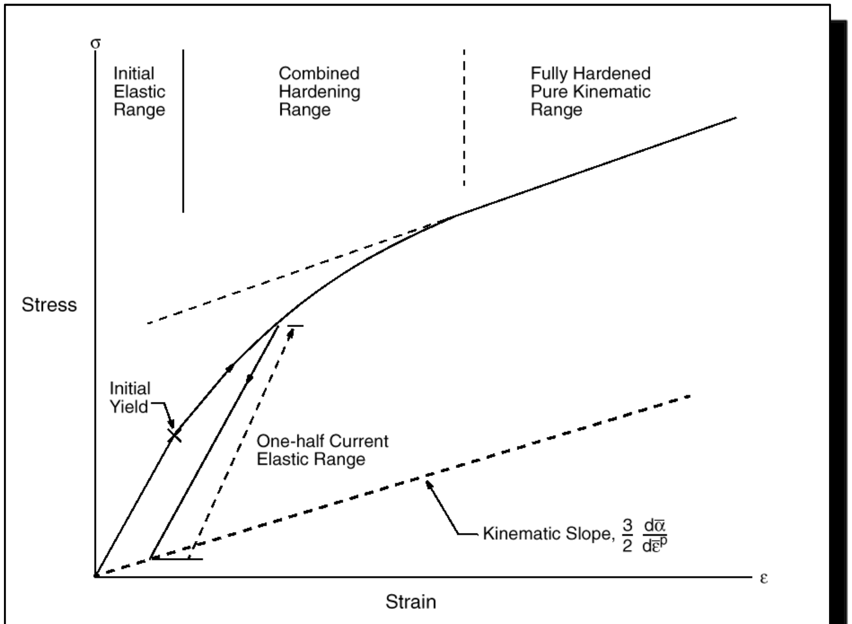


Figure b: Loading Path

Combined Hardening

- The initial hardening is assumed to be almost entirely isotropic, but after some plastic straining, the elastic range attains an essentially constant value, that is pure kinematic hardening.





Basic Uniaxial Tension Behavior of the Combined Hardening Model

Combined Hardening

- Combined Hardening is good for simulating the shift of the stress-strain curve apparent in a cyclical loading (hysteresis), either for cyclic hardening or cyclic softening.
- This is called the Bauschinger effect, which in Marc requires using Combined Hardening.
- If there is a shift of stresses with neither hardening or softening (the maximum stress in each cycle is the same), then the behavior is called ratchetting.
- A test often used to characterize the plastic behavior of metals is the Plastic Shakedown, essentially by producing symmetric strain cycles. Soft or annealed metals tend to harden towards a stable limit, and initially hardened metals tend to soften. These types of things show up in the Plastic Shakedown test.
- Another test is the Relaxation of Mean Stress. It appears on an asymmetric strain experiment, one in which the specimen is allowed to strain well into the plastic range before starting the load cycling.
  - As cycles increases, the mean stress tends to zero

Combined Hardening

- Combined Hardening implies a constant shift of the center of the elastic domain with a growth of elastic domain around this center until pure kinematic hardening is attained. In this model, there is a variable proportion between the isotropic and kinematic contributions that depends on the extent of plastic deformation - as measured by the mean plastic strain.
- The basic assumption of the combined hardening model is that such behavior is reasonably approximated by a classical constant kinematic hardening constraint with the superposition of initial isotropic hardening.
- The isotropic hardening rate eventually decays to zero as a function of the equivalent plastic strain measured by

$$\bar{\varepsilon}^p = \int \dot{\bar{\varepsilon}}^p dt = \int \left(\frac{2}{3} \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p \right)^{1/2} dt$$

- The work hardening data at small strains governs the isotropic behavior, and the data at large strains governs the kinematic hardening behavior.
- If the last work hardening slope is zero, the behavior is the same as the isotropic hardening model.