Calculation of Three-dimensional Positional Tolerances

Michael Radeck, Q-DAS® GmbH & Co. KG

In case you measure parts with a three-dimensional coordinate measuring machine, each measured value is defined by its x-position, y-position and z-position. Characteristics whose location in space is specified by three coordinates are referred to as "position characteristics". Engineering drawings often specify the tolerance for each axis separately (tolerance = permissible deviation from the nominal value). We refer to this type of tolerancing as one-dimensional tolerancing. However, the one-dimensional tolerancing is the wrong approach for multidimensional characteristics. First, we want to confirm this statement based on a transparent example including two dimensions. The distance from the nominal value = tolerance center to the upper specification limit is equal to the distance from the nominal value to the lower specification limit in the first dimension. However, in case the characteristic varies in two dimensions, the permissible distances from the nominal value to the respective specification limit suddenly differ. The nominal position is exactly in the middle of the tolerance square (see x/y-plot). If you measure the distance from the middle of the square to one of the corners of the tolerance square, you will find out that the distance is longer than the distance from the nominal value to the middle of one of the sides. As a result, the permissible deviations will differ considerably depending on the respective direction if you actually use the tolerance square. However, there is a solution to this problem. You have to use the tolerance circle whose radius complies with the range from the nominal value to the upper specification limit of the one-dimensional approach. The radius ensures that the maximum permissible distance from the nominal value is equal in all directions.

Example

The nominal position of a borehole is defined by $x_{nom} =$ 100 mm and $y_{nom} =$ 75 mm. The specification limits for the x-coordinate are $USL_x =$ 100.05 mm and $LSL_x =$ 99.95 mm whereas the specification limits for the y-coordinate are $USL_y =$ 75.05 mm and $LSL_y =$ 74.95 mm. Now we measure the borehole position of 50 parts. The following figure shows the one-dimensional measured values of each axis separately (in the value chart) and their two-dimensional dispersion (in the x/y-plot).

Measured values taken from a two-dimensional normally distributed population create a random dispersion ellipse.

The behavior in the first dimension can be translated into three dimensions. In this case, the tolerance is just a tolerance circle or, in general, a tolerance ellipsoid. When the measured values are taken from a normally distributed population, the values create a random dispersion ellipsoid whose shape resembles an American football.

Since version 11, you can use the Sample Analysis and Process Capability Analysis modules of the programs qs-STAT® and destra® to evaluate two-dimensional and three-dimensional positional tolerances correctly in a multivariate analysis according to ISO FDIS 22514 - Part 6. In case you do not transfer the measured values directly from the three-dimensional measuring machine to the program, you may only create positional tolerances by selecting File|New.
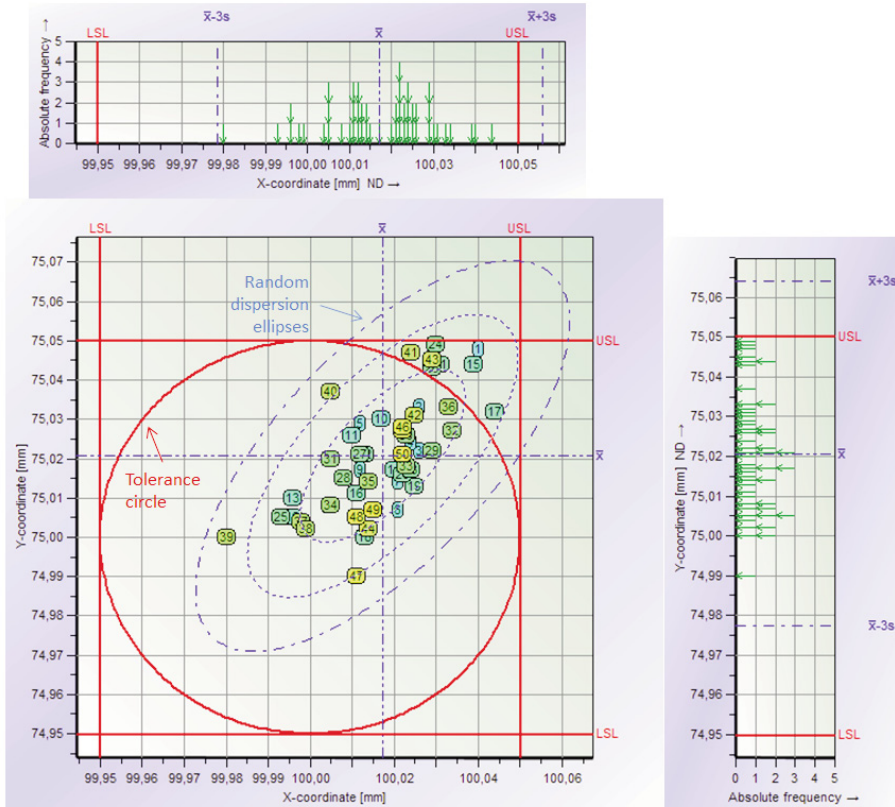
Positional tolerance: When regarding the one-dimensional display of values, all measured values seem to be just within the tolerance. However, the two-dimensional display shows that some measured values are outside the tolerance circle
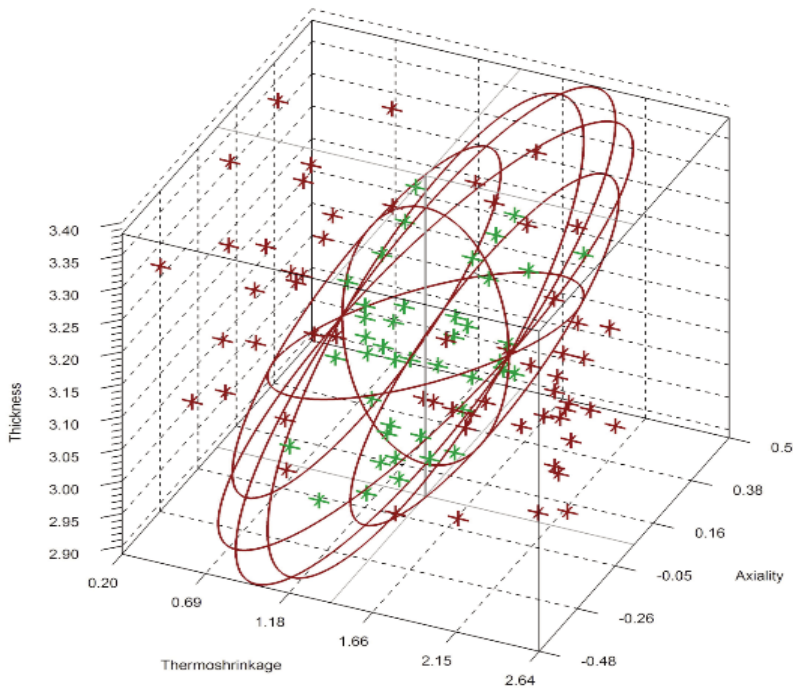
Three-dimensional random dispersion diagram including a tolerance ellipsoid and measured values. The green points of data are within the tolerance ellipsoid, the red values are outside the tolerance.

Calculation of Capability Indices P₀ and P₀ₖ

The program calculates the respective three-dimensional positional tolerance based on the x-coordinate, y-coordinate and z-coordinate of the entered specification limit. A normal distribution including a three-dimensional random dispersion is adapted to the measured values. The calculation of capability indices described in the following is based on the international ISO FDIS 22514-6 standard.

Calculating capability index P₀

1. You shift the location of the normal distribution by moving the expectation of the 3-D normal distribution to the nominal position: $\mu_x = X_{nom}$; $\mu_y = Y_{nom}$ and $\mu_z = Z_{nom}$.
2. You calculate the volume (= probability) $P_{P_0} = V_{P_0}$ of the dispersion ellipsoid of the 3-D normal distribution whose surface area just touches the surface area of the tolerance ellipsoid. This value indicates the probability of gaining measured values within this tolerance ellipsoid when shifting the process exactly to the nominal position.
3. Now you calculate quantile z based on the inverse distribution function of the one-dimensional standard normal distribution Φ^{-1}: $z = \Phi^{-1}[(P+1)/2]$.
4. The last step is to calculate the capability index P₀ by dividing quantile z by 3: $P_0 = z/3$

Calculating capability index P₀ₖ

The calculation of the minimum capability index P₀ₖ corresponds to steps 2 to 5 of the P₀ calculation. The only difference is that you do not take the first step, i.e. shifting the 3-D normal distribution to the nominal position. The 3-D normal distribution adapted to the measured values thus remains in the estimated process location

$$\mu_x = \bar{x}; \quad \mu_y = \bar{y} \quad \text{und} \quad \mu_z = \bar{z}$$

Field of application

The assembly sector mainly applies 3-D positional tolerances, e.g. while installing lightning units into a car. As an example, the coordinates of selected dimension reference points define the nominal position of a headlight. The actual position of the points is measured and specified in the form of 3-D coordinates. Due to the calculation method described in this article, you are able to evaluate the capability referring to the positions of the reference points.
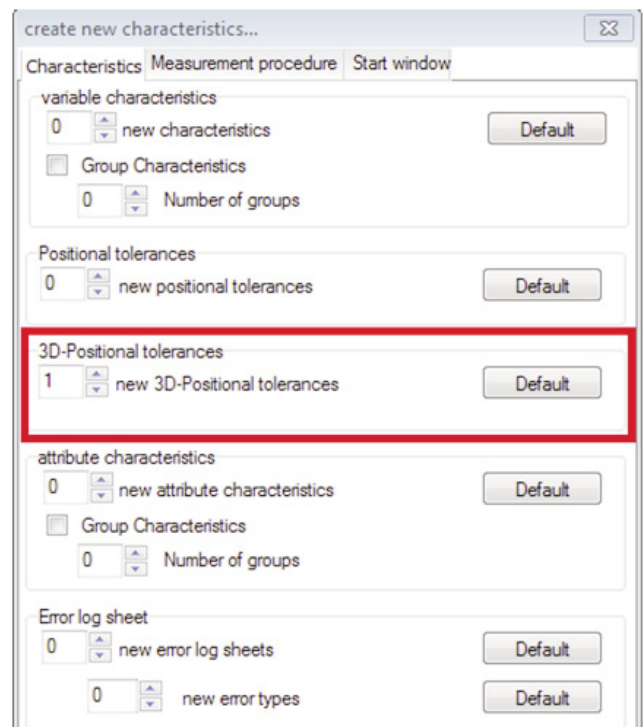
Manual input of a 3D positional tolerance in the Sample Analysis and Process Capability Analysis modules of the programs qs-STAT® and destra®