Operation characteristics Quality control chart

qs-STAT process Capability

FAQ
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Document History

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CONTENTS

1 Overview

We consider in this document the graphical representation of the operation characteristic function for quality control charts. The graphic operation characteristic for a location chart shows us at which displacement of the location parameter μ a violation of control limit will occur in the location chart and with what probability.

Similarly, the graph of the operation characteristic function for a variation chart shows us how much the probability of obtaining a violation of control limit increases when the value of the variation parameter σ has increased.

With the operation characteristic, we users can thus determine how sensitive a control chart is to an undesired change in the observed parameter - either μ or σ. This information helps us users to select a suitable quality control chart for the purpose of process monitoring and control.

2 Operation characteristics of the SPC location chart

A quality control location chart, such as a Shewhart average chart, is used to monitor the location parameter μ of the process distribution. If the process is to be guided to a target value, a process operator wants the location parameter of the process distribution not to change. Unfortunately, we users cannot measure and observe the location parameter μ directly. But with the help of subgroup statistics and values calculated from them, such as the sample mean, we can at least estimate the current value of the location parameter. Sampling results are always subject to random variation. From the pattern of the sample mean values in the control chart, see whether these values show a common cause variation pattern around a constant parameter μ, or whether the values show a special cause pattern like a location shift or a trend.
But how sensitive is a chosen control chart to a shift of the location parameter μ? The answer to this question is the operation characteristic curve.

In the following, a combination of a Shewhart average chart and a Shewhart standard deviation chart is shown using the example data set Test *2* from the test file *TEST_ALL.dfq.* Thus, if the data set *Test 2* is evaluated with the evaluation strategy "*Q-DAS Process Capability (01/2020)*", a combination of a Shewhart average chart and a Shewhart standard deviation chart opens in the programme when the SPC quality control chart is called up for this data set. After loading the file *TEST_ALL.dfq* into the module *Process Analysis of* the programme *qs-STAT,* first select the characteristic *Test 2* in the list of characteristics and then select the menu command

Start | *Evaluation Strategy* | *Q-DAS Process Capability (01/2020)*

The control chart graphic is opened in the programme with the following command:

Graphics | *Control Chart* | *SPC QCC* | *Representation 1*

The control chart can now be seen in the *SPC QCC - Display 1* window. The following figure shows an example of the SPC control chart:

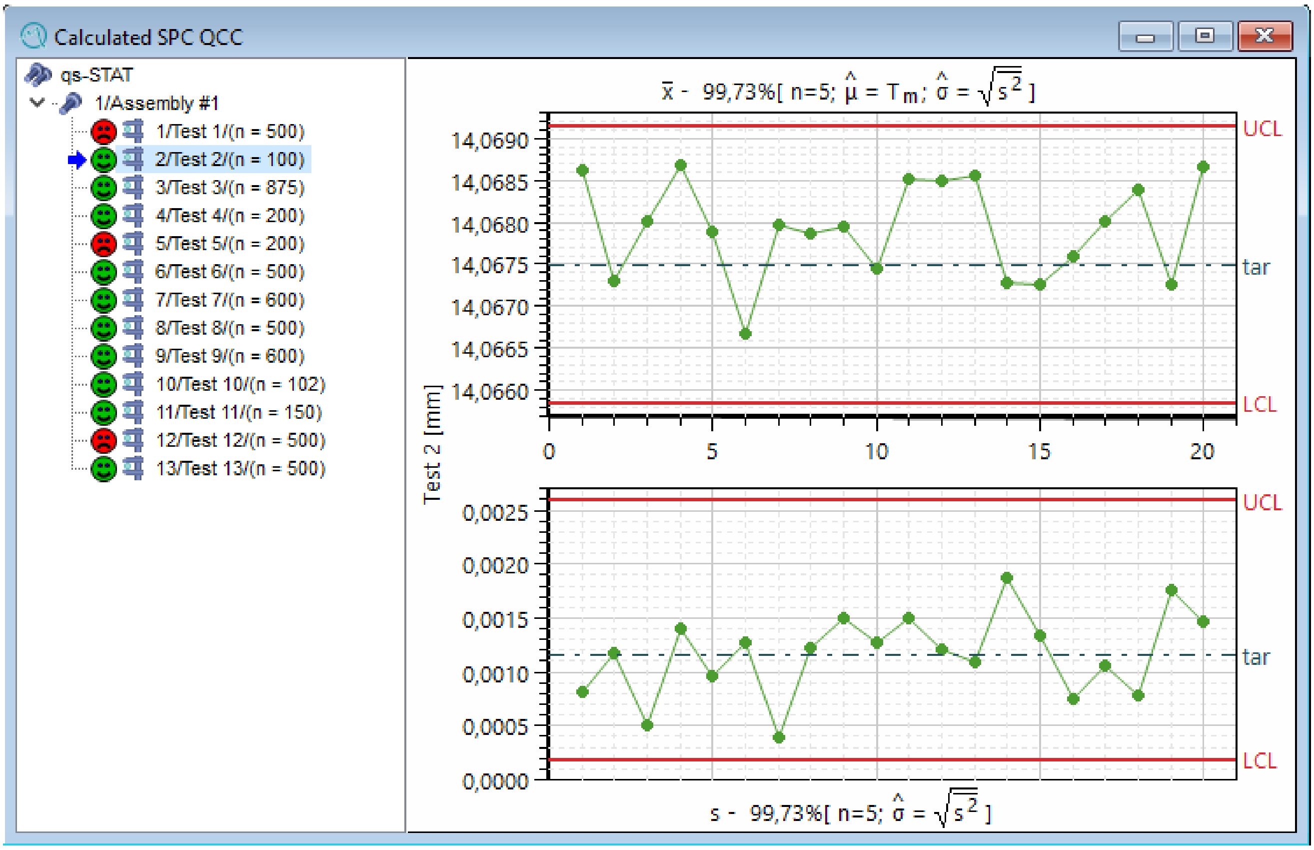


*Figure 1: View of the SPC QCC for the characteristic **Test 2** from the data set **TEST_ALL.dfq**. The SPC QCC is a combination of a Shewhart average chart for monitoring the process location parameter μ and a Shewhart standard deviation chart for monitoring the process dispersion parameter σ.*

With the command ...

Graphics | *Control Chart* | *Operation Characteristics*

...open the graphic Operation Characteristics Analysis QCC. As soon as the window is open, select the command:

Graphic settings | *2. QCC on/off* | *SPC QCC*

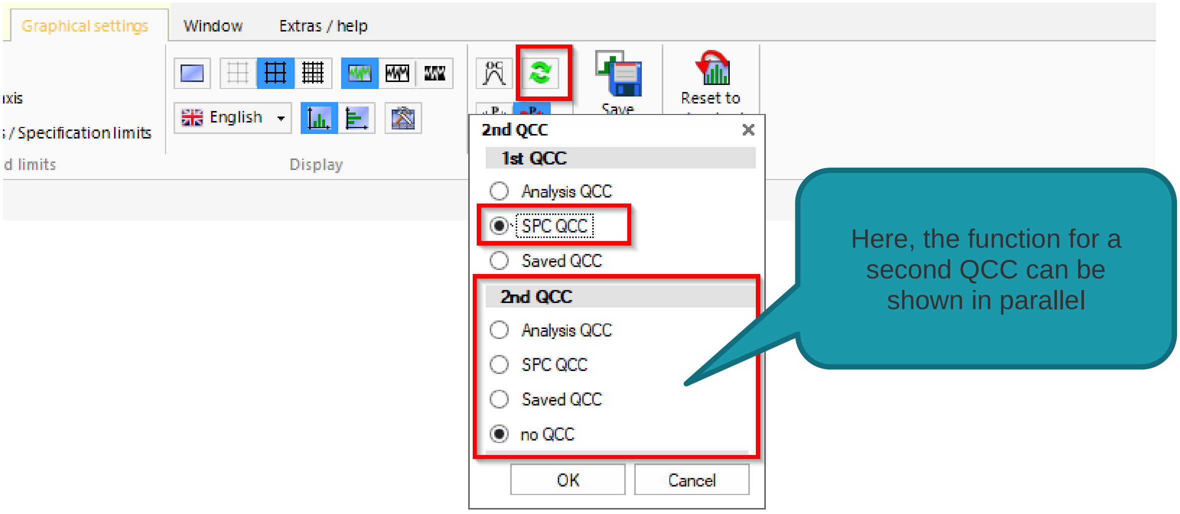


Figure 2: Clicking sequence for selecting the SPC QCC display in the Operation Characteristics window.

2.1 Non-interference probability β (operation characteristic)

To display the graph of the operation characteristic (*non-interference probability β* [1]), click the command:

Graphic Settings | *Display* | *Non-interference Probability*

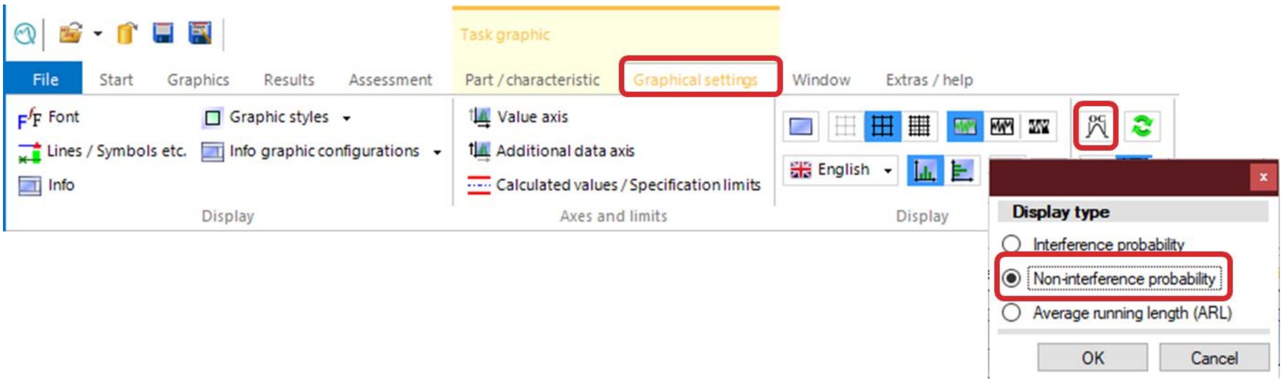


Figure 3: Selection of the display type "non-interference probability" for the operation characteristic graphic

[1] In the context of statistical hypothesis testing, this probability is often referred to as the probability of the 2nd kind, or as the β-risk. The value for β indicates the probability of no violation of the control limits occurring in a QCC, even though the current parameter value for µ or σ has changed in comparison to the parameter originally used for the construction of the QCC. μ_0 or σ_0 used for the construction of the QCC.

In the window *Operation characteristic Analysis QCC, the* function of the *non-interference probability* β for the location chart (Shewhart average chart) of the SPC QCC can be seen.

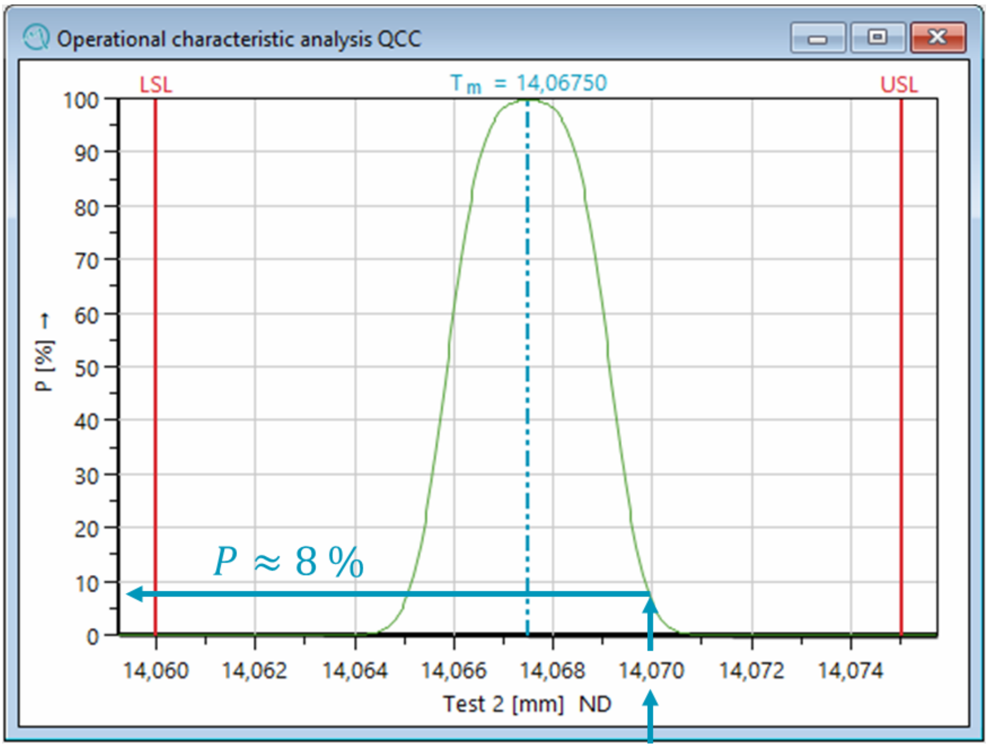


Figure 4: Graph of the non-interference probability β (P in the figure) for the Shewhart average chart (SPC QCC), modified for the characteristic Test 2 of the data set TEST_ALL.dfq.

The number line on the **X-axis can** be imagined as **any potential location of the parameter μ of the process distribution**. The left red line symbolises the location of the lower specification limit $LSL = 14{,}060\ mm$ and the red line on the right symbolises the location of the upper specification limit. $USL = 14{,}075\ mm$. The Shewhart average chart (SPC QCC) shown here was originally constructed with the location parameter ...

$$\mu_0 = \frac{USL + LSL}{2} = 14{,}0675\ mm$$

In this case, this is exactly the middle of the tolerance. The dispersion parameter σ is assumed to be constant in this graph, so it does not change its value. On the (left) Y-axis is the *non-interference probability* β (P), i.e. the probability that the control limits will **not be** exceeded.

The graph should be read as follows: If the process parameter μ is exactly at the value μ_0 (in the figure this is the middle of the tolerance T_m), the highest possible value for the non-interference probability is obtained: $1 - \alpha = 99{,}73\ \%$. This is what the chart was constructed for. If now, for example, the location parameter moves from the centre of tolerance $\mu_0 = T_m = 14{,}0675\ mm$ to the value $\mu_{disturbed} = 14{,}070\ mm$ the next subgroup drawn in the Shewhart average chart would, with a probability of approx. $\beta = 8\ \%$ **no violation** of control limit would be observed (*non-interference probability* β).

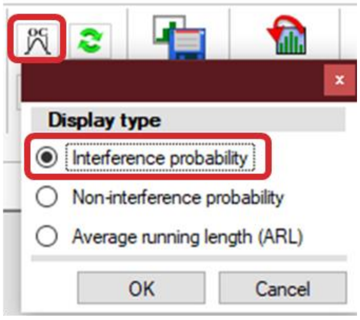
The OC function expresses the risk of not obtaining a violation of control limit with the next drawn subgroup, even though the location parameter has shifted.

2.2 interference probability $1-\beta$

The probability complementary to the non-interference probability β-risk is the *interference* probability. [2]$1-\beta$. With reference to the Shewhart location chart, the interference probability expresses the probability of observing a violation of control limit when the parameter µ is shifted in the QCC.

To see the interference probability function, click on the commands in the menu:

Graphic Settings | *Display* | *Interference Probability*



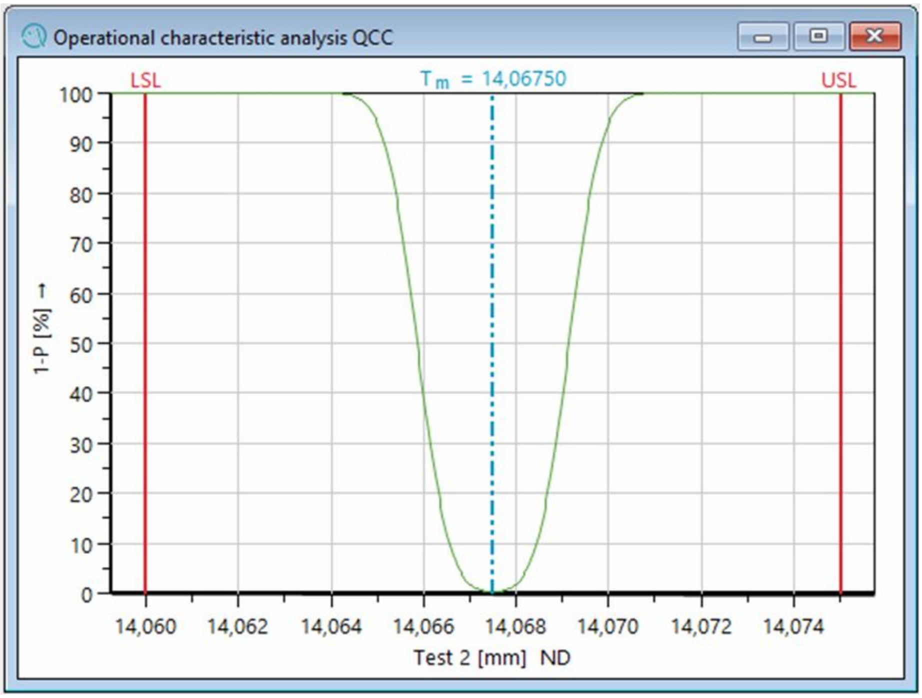


Figure 5: Graph of the interference probability $1-\beta$ *for the Shewhart average chart (SPC QCC) based on the data set "TEST_ALL.DFQ", characteristic "Test 2".*

[2] In the field of statistical hypothesis testing, the *interference probability is usually referred to as* $1-\beta$ is usually referred to as *power.* This probability expresses how "certain" a violation of control limit occurs.

2.3 Expected proportion outside specification

The following command displays the expected proportion outside specification limit (expected proportion of characteristic values outside specification limit) depending on the process location µ:

Graphic Settings | *Show proportion outside specification*

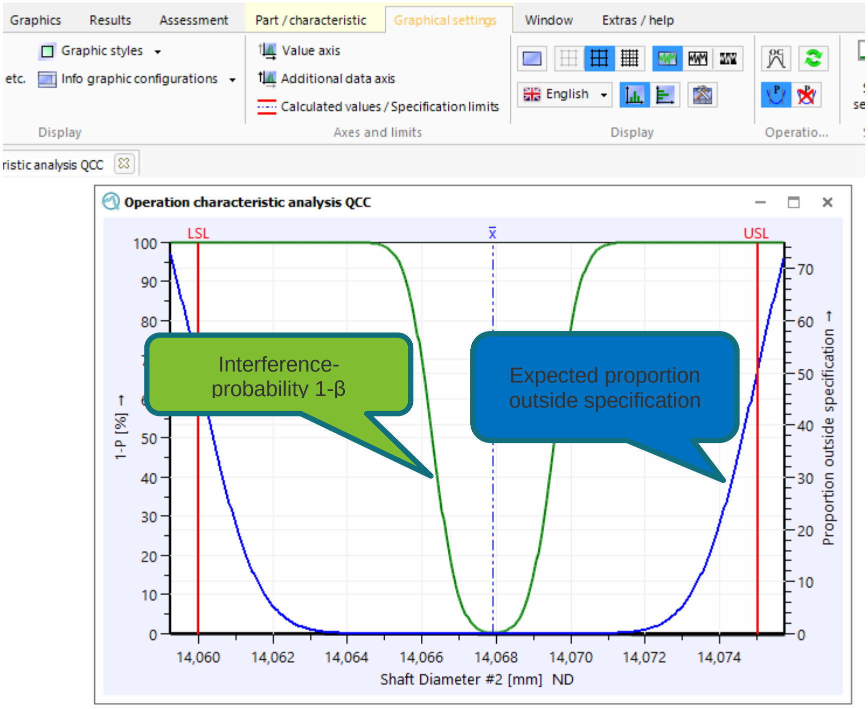


Figure 6: Expected proportion outside specification as a function of the value of the process location parameter µ. Note the right Y-axis for the proportion outside specification and the left Y-axis for the interference probability. $1 - \beta$.

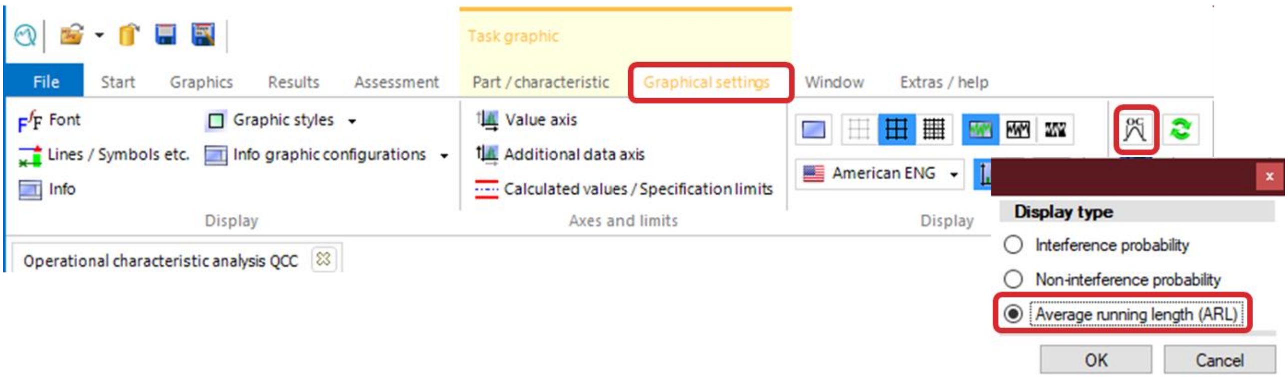
The following applies to the test data set *Test 2*: In the Shewhart average chart, after a process location shift to $\mu_{disturbed} = 14{,}070\ mm$ there is a high probability (approx. 92 %) that a violation of control limit would be observed even before a serious proportion outside specification would have occurred. This is a desirable situation! But unfortunately, this is only given for those processes for which a high process capability could be proven (e.g. if Cp and Cpk are both 2.0 or greater).

2.4 Average Run Length (ARL)

With the menu command ...

Graphic Settings | *Display Type* | *Average Run Length (ARL)*

... the graph of the average run length is displayed.



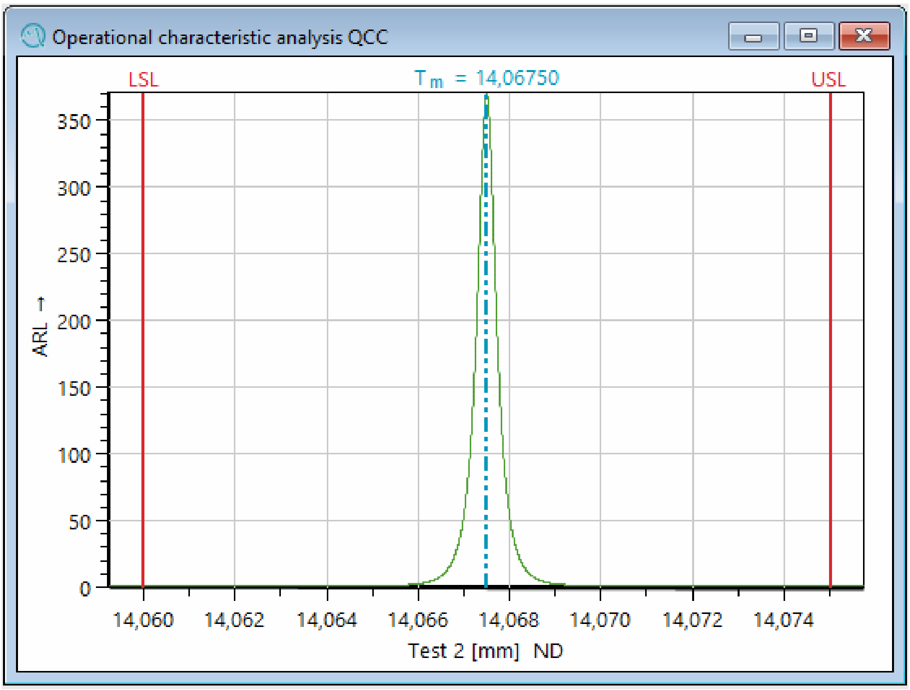


Figure 7: Graph of the average run length as a function of the location parameter µ.

The mean run length ARL shows us users for a (arbitrarily chosen) value µ how many samples would have to be drawn on average until a violation of control limit will occur in the Shewhart average chart. For the chart centred exactly on the tolerance centre, it takes an average of approx. 370 sample drawings (mountain peak) until a violation of one of the control limits occurs. For the location parameter $\mu_{distubed} = 14{,}068\ mm$ it takes an average of about 50 samples before a violation of control limit occurs and for the subgroup $\mu_{disturbed} = 14{,}069\ mm$ it is less than 10 subgroups.

3 Planning the operation characteristics

The standard settings for the quality control charts in the programme are taken from the currently active evaluation strategy This also applies to the graphic of the operation characteristic, which is displayed either for the analysis location chart or for the SPC location chart of the evaluation strategy. However, the qs-STAT programme also offers us users a planning tool for the operation characteristics of a quality control chart that is independent of the evaluation strategy. To do this, we select the following command in the menu bar:

Supplements / Help | *OC Planning*

In the *Operation Characteristics* window, QC curves are available for the following chart types:

chart type	Card type	Symbol
Shewhart location chart	Shewhart average chart	$\bar{x} - chart$
	Shewhart median chart	$\tilde{x} - chart$
	Shewhart Raw values chart	$x - chart$
Acceptance location chart	Acceptance chart	$\bar{x} - ACC$
	Median chart	$\tilde{x} - ACC$
	Raw values chart	$x - ACC$
Shewhart variation chart	Exactly calculated Shewhart standard deviation chart	$s - chart$
	Approximated Shewhart standard deviation chart	$s - chart$ (QS-9000)
	Exactly calculated Shewhart range chart	$R - chart$
	Approximately calculated Shewhart range chart	$R - chart$ (QS-9000)

Notes:

1. With the "exactly calculated standard deviation chart", the control limits are calculated with the $\frac{\alpha}{2}$ or the $1 - \frac{\alpha}{2}$ quantile of the χ^2-distribution, and for the "exactly calculated range chart" the control limits are determined using the $\frac{\alpha}{2}$ or the $1 - \frac{\alpha}{2}$ quantile of the w-distribution. $\left(w = \frac{R}{\sigma}\right)$. The asymmetric dispersion pattern behaviour of the sample parameters s and R around their respective expected value is thus correctly described (for samples from a normally distributed subgroup).

2. For the "approximately calculated standard deviation chart" and for the "approximately calculated Range chart", the control limits are approximated with the $\frac{\alpha}{2}$ or the $1 - \frac{\alpha}{2}$ quantile of the normal distribution. The asymmetric dispersion pattern of the two sampling statistics s and R around their respective expected value is thus not correctly described.

3.1 OC planning for a Shewhart location chart

The handling and interpretation of OC planning is described using the Shewhart average chart as an example. The procedure is the same for the other two Shewhart location charts.

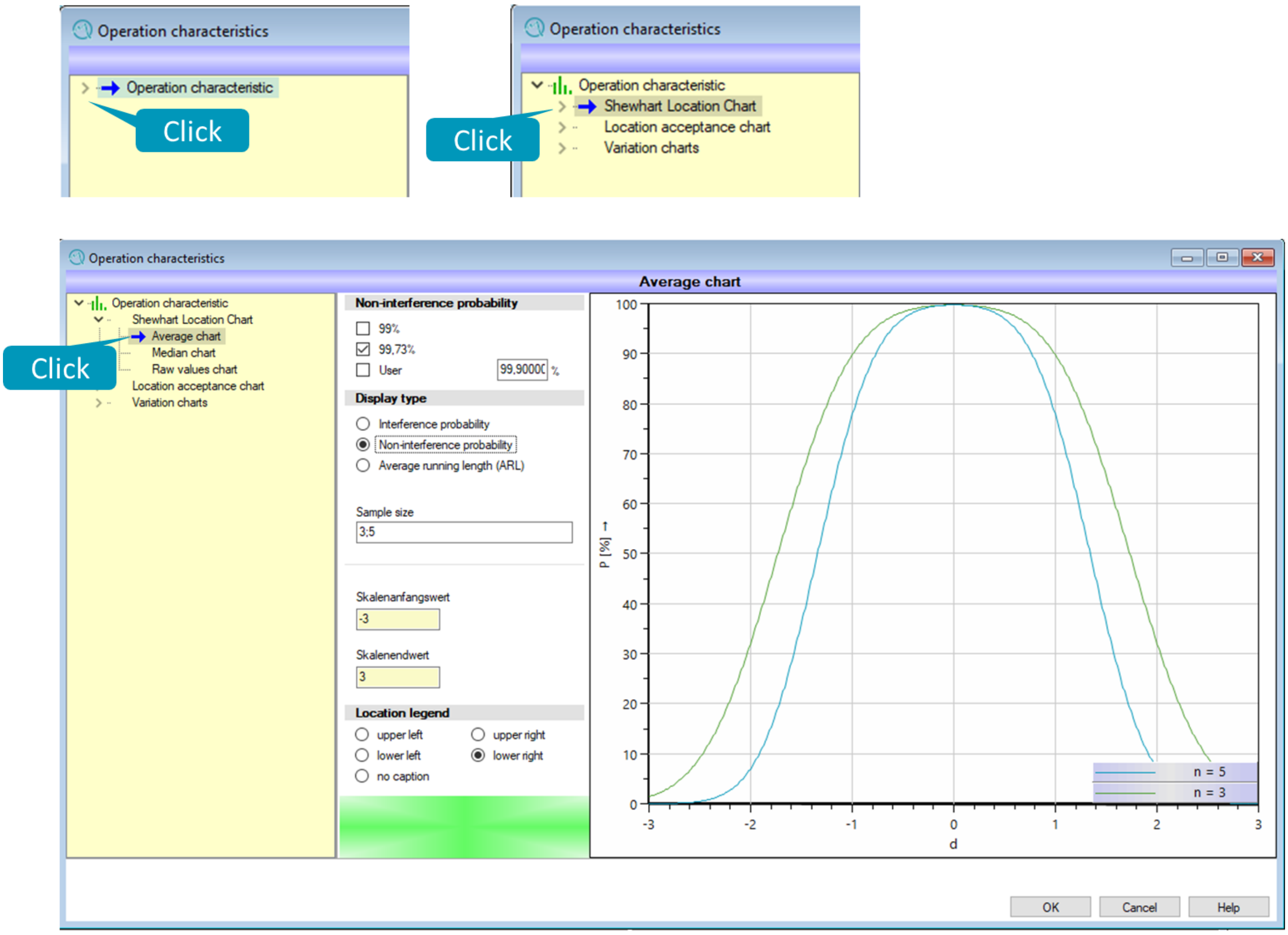


Figure 8: The operation characteristic planning window shows the non-interference probability function of a Shewhart average chart in comparison for the construction of the chart with sample size n = 3 (green) and n = 5 (blue).

Option Non-interference probability

Under the option *non-interference probability* β the user selects the probability for the construction of the chart. Internationally common is the *non-intervention probability* $\beta = 99{,}73\ \%$ according to *Walter A. Shewhart*. In Europe, due to the influence of British standards, the probability of non-interference $\beta = 99\ \%$ is still often used.

If required, the user can set a user-specific value for the *non-interference probability*, such as $\beta = 99{,}9\ \%$ (or any value in the interval $50\ \% < \beta < 100\ \%$).

Display type option

The programme offers the following function display types:

- Interference probability 1-β
- Non-interference probability β
- Average running length ARL

Option subgroup size

Different subgroup sizes can be entered here, each separated by a comma or a semicolon. This allows the influence of the subgroup size on the shape of the currently selected representation function to be examined and compared.

Start of scale value and end of scale value option

The value interval shown on the X-axis is defined here. On the X-axis, the positional shift of the average d is shown in standardised form. This means that the location shift is expressed in units of the standard deviation σ. For the location chart, the value $d = 1$ means that the location parameter μ is shifted by one standard deviation from the original position μ_0. The value $d = 2$ means that the location parameter μ is shifted by two standard deviations from the original position, and so on. μ_0 from the original position, etc.

Option Legend position

Here the user decides whether and where the legend should be displayed on the graphic.

Clicking on the *OK* button triggers the calculation and display. The procedure and interpretation for *the Shewhart median chart* and *Shewhart raw values chart* is analogous.

3.2 OC planning for an acceptance quality control chart

In contrast to a Shewhart location chart, the acceptance chart is an option when the location parameter of a process cannot be kept constant. Let's think here, for example, of tool-based manufacturing processes such as injection moulding, broaching, reaming, drawing, etc. In all these processes, a change in position due to tool wear is unavoidable and the tool should be used for an "optimally long" service life for economic reasons. This makes the thought of a setpoint control with a Shewhart location chart superfluous. Instead, the expected proportion outside specification p is considered. The user enters the values for the “current proportion of defective units” and the “Inference Probability”.

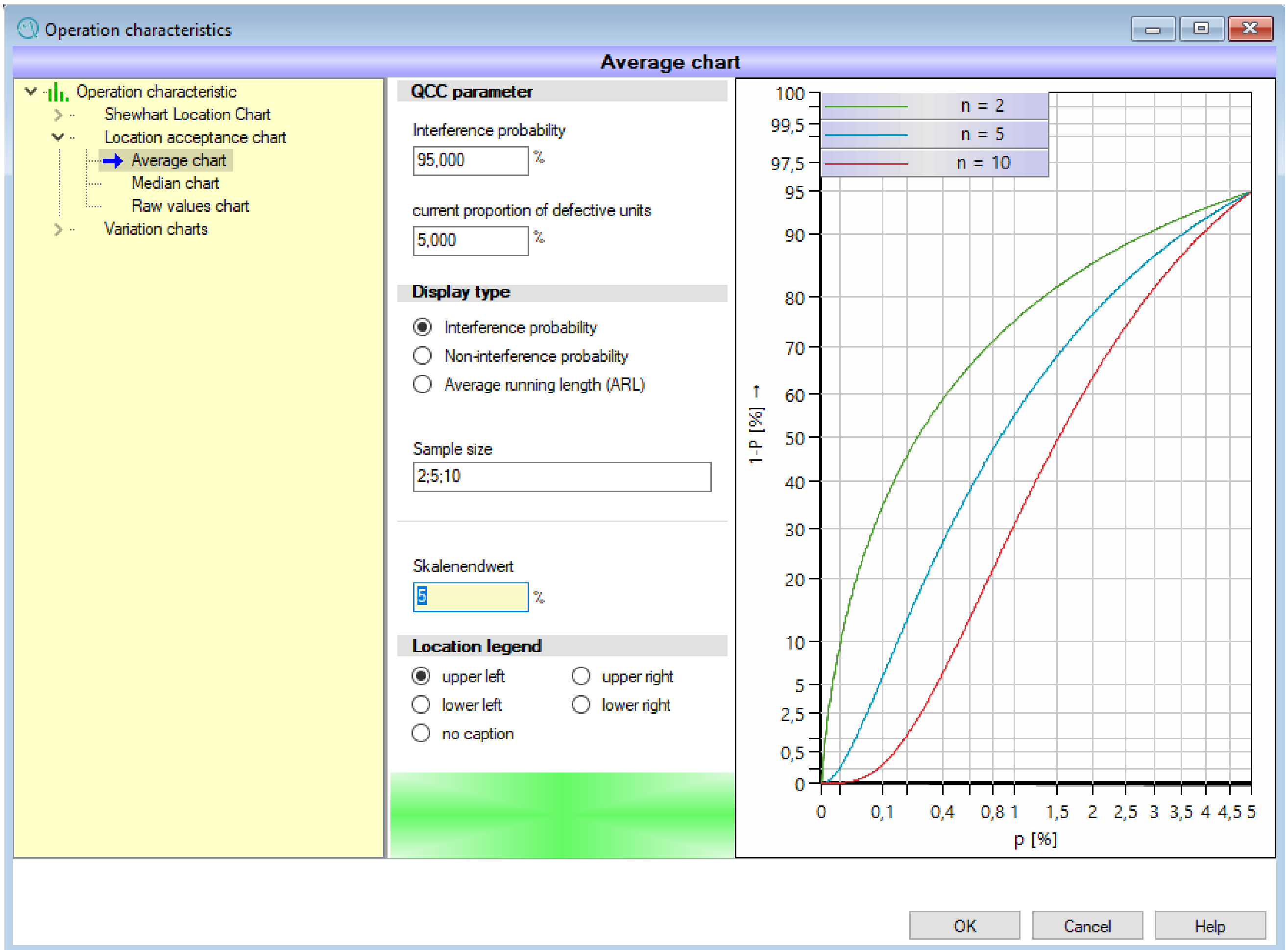


Figure 9: Intervention curves for an average assumption map in comparison for the subgroup sizes n = 10, 5 and 2.

The intervention characteristic curve in Figure 9 is read as follows: If the current proportion outside specification has increased to p = 1 % (X-axis), an limit violation will occur with about 30 % probability in the next drawn subgroup in the mean outside specification map with sample size n = 10. In the average acceptance chart for the subgroup size n = 5, a violation of control limit is obtained with approx. 55 % and for the subgroup size n = 2, a violation of control limit with 75 %.

3.2.1 Options QCC parameters

To construct the chart, the user must fill in the following two input fields:

1) The interference probability $1 - \beta$ in percent

2) The proportion outside specification to be "safely" detected $p_{Default}$ in percent

3.2.2 Display type option

Here, the user can first select one of the three display types ...

(1) interference probability $1 - \beta$

(2) Probability of non-interference β

(3) Average Run Length (ARL)

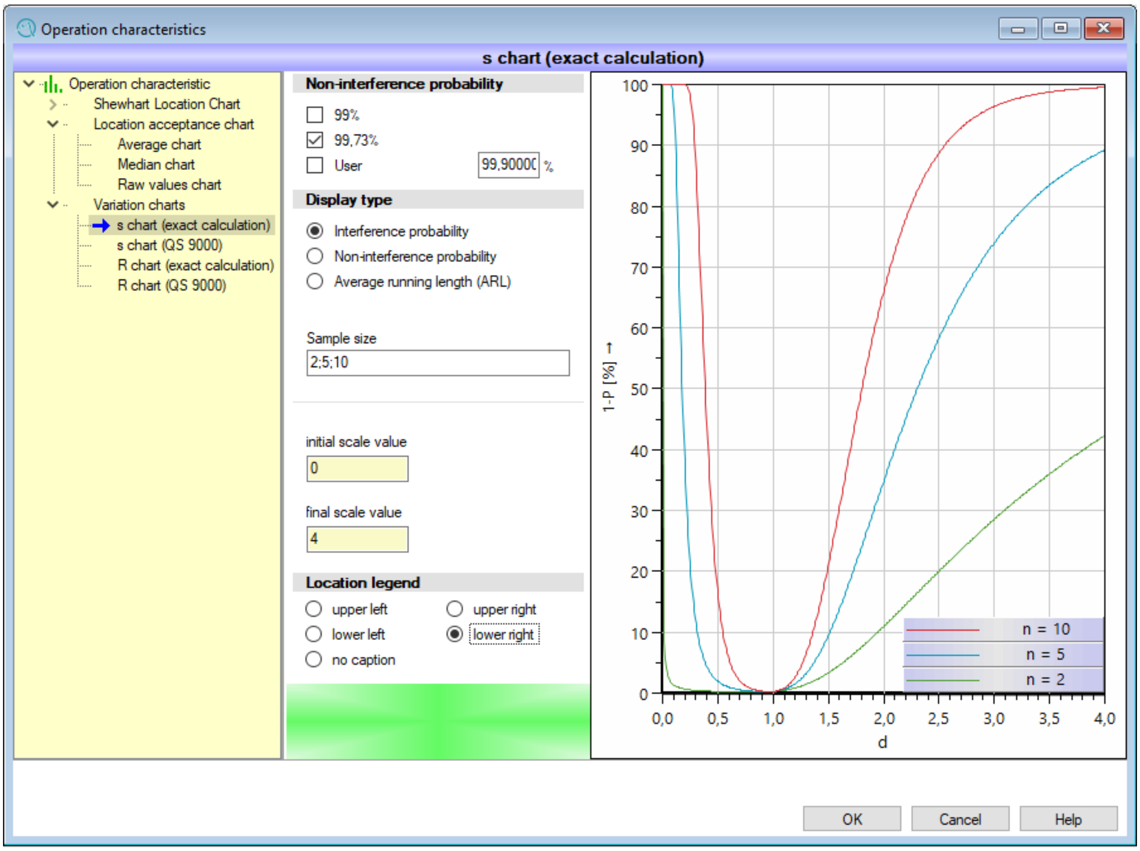
In the "Subgroup *size*" input field, the user can enter several sample sizes, each separated by a comma or a semicolon.

The input field "*Scale end value*" refers to the display length of the X-axis, i.e. the proportion outside specification p. For the proportion outside specification, the user can enter a value in the interval between 0.05 % and 100 %.

3.3 OC planning for a Shewhart variation chart

For the variation charts, operation characteristics can be calculated for the Shewhart standard deviation chart (s chart) and for the Shewhart range chart (R chart).

In Figure 9 the intervention curves of the Shewhart standard deviation chart for the subgroup sizes n = 10, n = 5 and n = 2 are shown for comparison. On the X-axis the ratio $d = \frac{\sigma_{actual}}{\sigma_0}$ is plotted as a number line, i.e. the ratio of the current value for the parameter σ to the original value $\sigma_0$with which the chart was constructed. Thus, d = 2.0 means that the current value for sigma is twice as large as the sigma value used to construct the Shewhart s chart. It is worth noting that a change in the sigma value is comparatively hard to notice with a small subgroup size. A doubling of the sigma value (d = 2) only leads to an interference limit violation with a probability of 12 % for the next subgroup size n = 2. In comparison, note the characteristic curve for the subgroup size n = 10. Here, the interference probability is already around 68 %.



The interpretation is analogous for the other variation charts. Note, however, that for the approximately calculated standard deviation chart (s chart QS-9000) there is no lower[3] control limit for a subgroup size smaller than n = 6. The same applies to the approximately calculated range chart (R chart QS-9000) for a sample size smaller than n = 7. This leads to the fact that for these sample sizes the associated characteristic curves for the intervention and non-interference probability are in each case not shown on the left or below d = 1.

[3] This applies to the construction of the s or R chart with a non-interference probability of 99,73 % according to *W. A. Shewhart*.