Position Deviations

Create and Evaluate

FAQ
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CONTENTS

Preface

In this document we deal with two-dimensional position deviations. We do not consider the one-dimensional position deviations for lines and planes.

In the first chapter we start with basics. Using an example, we look at how a Requirement for a position deviation is represented in a technical Drawing and how it is to be interpreted. We also clarify the question of how we get from the position measurement results to the values of the position deviation amounts.

In the second chapter we deal with the handling, i.e. how a positional feature is created in the programme qs-STAT, how the Positional tolerances are entered and how we carry out the evaluation.

The calculation details for determining the Capability index can be found in the third chapter.

Note: For the evaluation we have to consider that the calculation options for the Positional tolerances in the evaluation strategy can be set differently depending on the company. It is also possible that an evaluation strategy adjusted in this way allows for none calculation at all. Should this apply to the personal situation of a reader, he/she will also receive none evaluation results for the position deviations.

The company Q-DAS supplies the programmes qs-STAT and destra in the module Sampling Analysis with the evaluation strategy "Q-DAS Machine Capability (06/2013)" and in the module Process Analysis with the

evaluation strategy "Q-DAS Process Capability (06/2013)" (as of spring 2016). Both strategies include calculation and capability assessment.

The present case study was developed with the programme qs-STAT to a large Part in the module

"Sample Analysis" with the evaluation strategy "Q-DAS Machine Capability (06/2013)". In part, temporarily modified evaluation strategies were used in order to be able to adjust the calculation options for the position deviations. The sections affected by this contain corresponding notes.

1 Two-dimensional position deviations

In this chapter we will look into the question of what a position deviation is. Let us assume the following excerpt from a technical Drawing (as a non-standard sketch) related to the position of a borehole.:

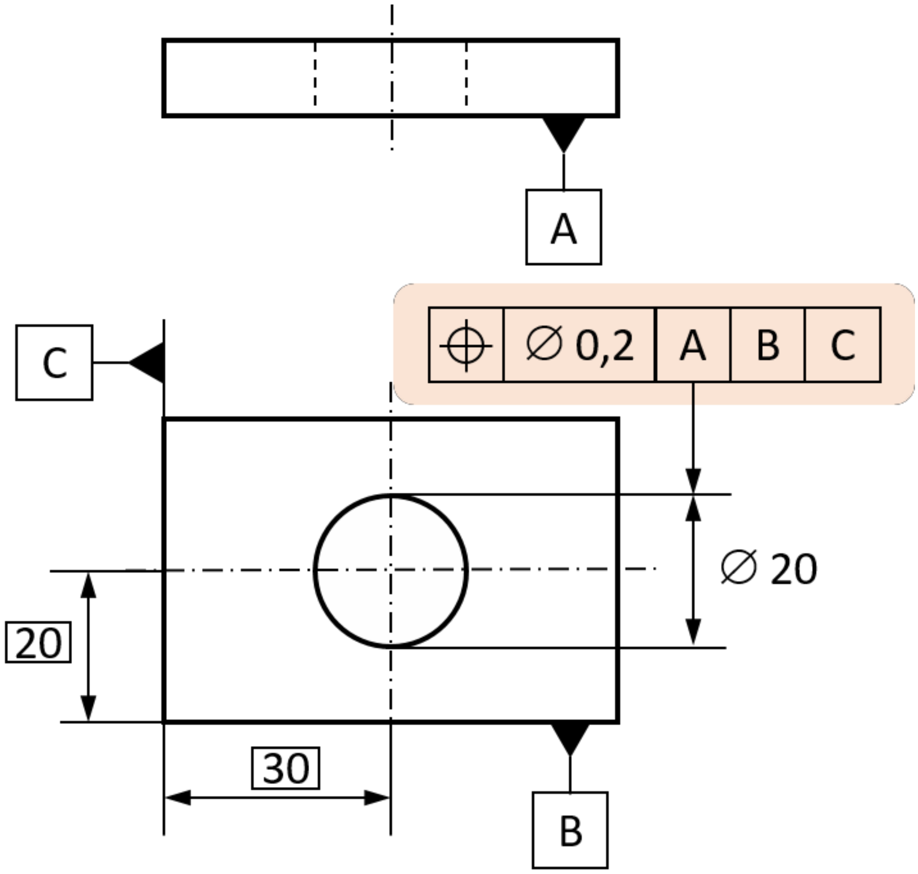


Figure 1-1: Sketch of a toleranced position for a hole in a drill plate

From the sketch we can see that the position of the hole - i.e. the centre of the hole - has been toleranced. The designer has provided that for manufacturing and metrological position determination the primary reference is edge A, the secondary reference is edge B and the tertiary reference is edge C.

The *diameter symbol* ∅ in front of the *Positional tolerances* $t_{PS} = 0{,}2\ mm$ states that the positional deviation in the plane may occur radially in any direction. However, our definition is still incomplete, because the tolerance here is three-dimensional: The tolerated position deviation applies to the total length of the bore, i.e. into the depth of Figure 1-1
.

Figure 1-2 shows the **theoretically exact nominal centre axis of the bore** as the **intersection of the two symmetry planes drawn in green**. Around this ideal position of the centre axis, the **Tolerance cylinder with the diameter t_{PS} is drawn in red.** As long as the centre axis of the bore lies within the surface of this red Tolerance cylinder over its entire length, it is a permissible position deviation.

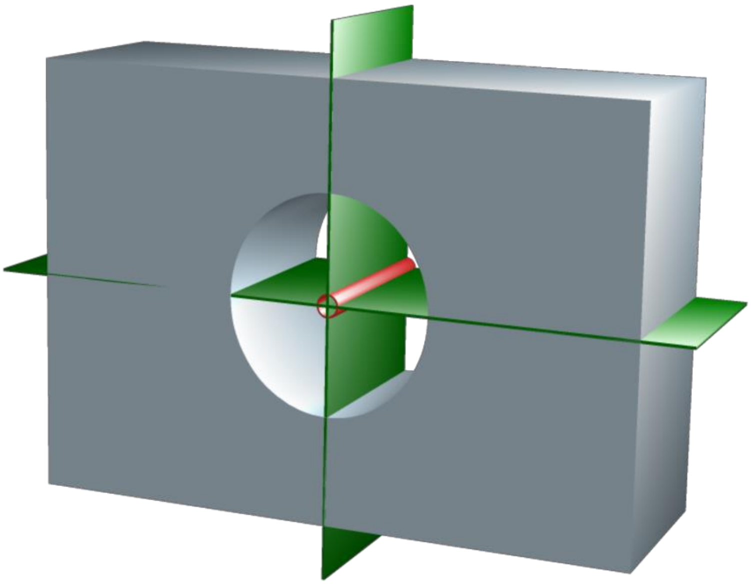


Figure 1-2: Illustration of the Positional tolerances as a tolerance cylinder (red) with diameter t_{PS}

As a rule, we receive from the Measurement system as the measurement result of a position calibration measurement only the X- and Y-coordinate of the actual position (x_{act} | y_{act}) to the largest measured position deviation, i.e. without the indication of the depth information (here: Z-coordinate). Here we have tacitly assumed that a Measurement system for determining the position deviation actually carries out several measurements at different depth levels of the borehole, but only outputs the one result of the maximum deviation as 2D information.

Since the depth information is omitted, we determine the position deviation as the difference between the actual and target position with the vector calculation in the plane

1.1 Vector of the target position

The nominal position of the hole is taken from the Drawing and has the following point coordinates in our numerical example:

$$\vec{P}_{tar} = \begin{pmatrix} x_{tar} = 30{,}00\ mm \\ y_{tar} = 20{,}00\ mm \end{pmatrix}$$

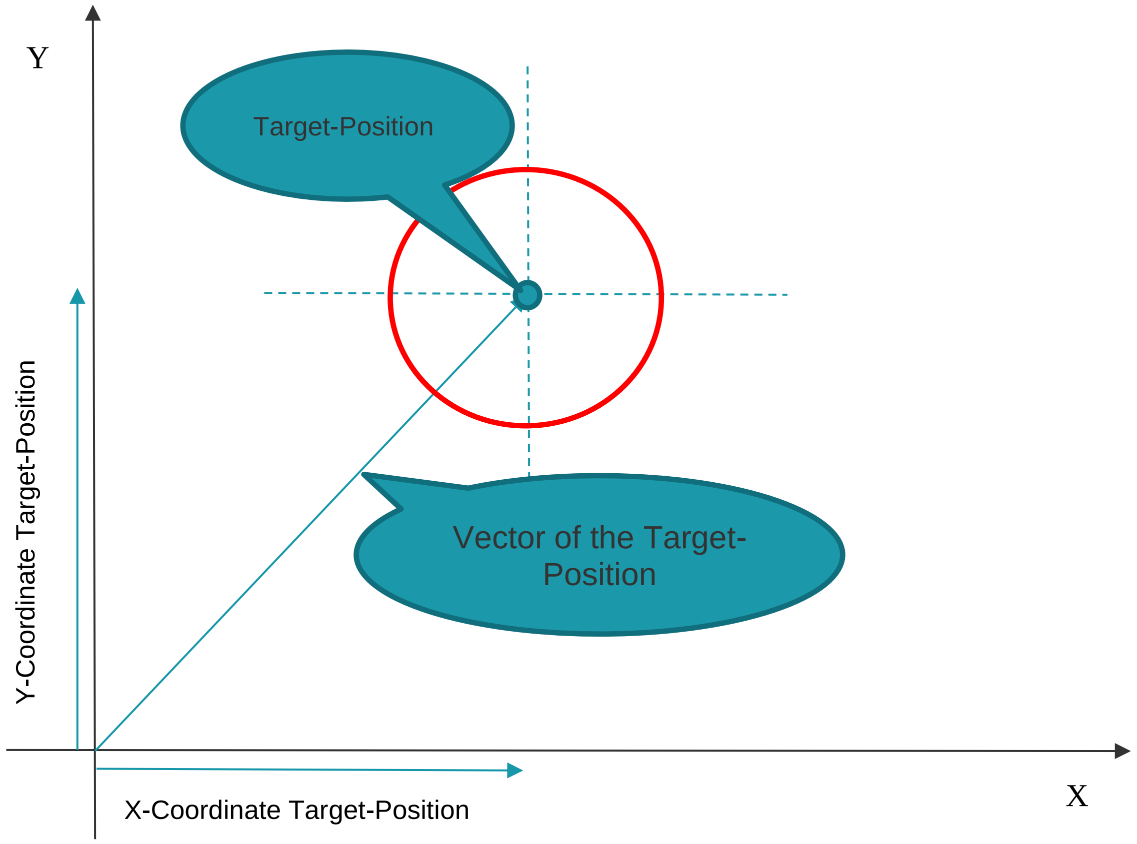


Figure 1-3: Sketch not drawn to scale to illustrate the vector for the target position, with the tolerance circle for the position deviation drawn in red.

Now a drill plate was measured…

1.2 Vector of the actual position

Let the measurement result for the coordinates of the actual position be the vector:

$$\vec{P}_{curr} = \begin{pmatrix} x_{curr} = 30{,}05\ mm \\ y_{curr} = 20{,}04\ mm \end{pmatrix}$$

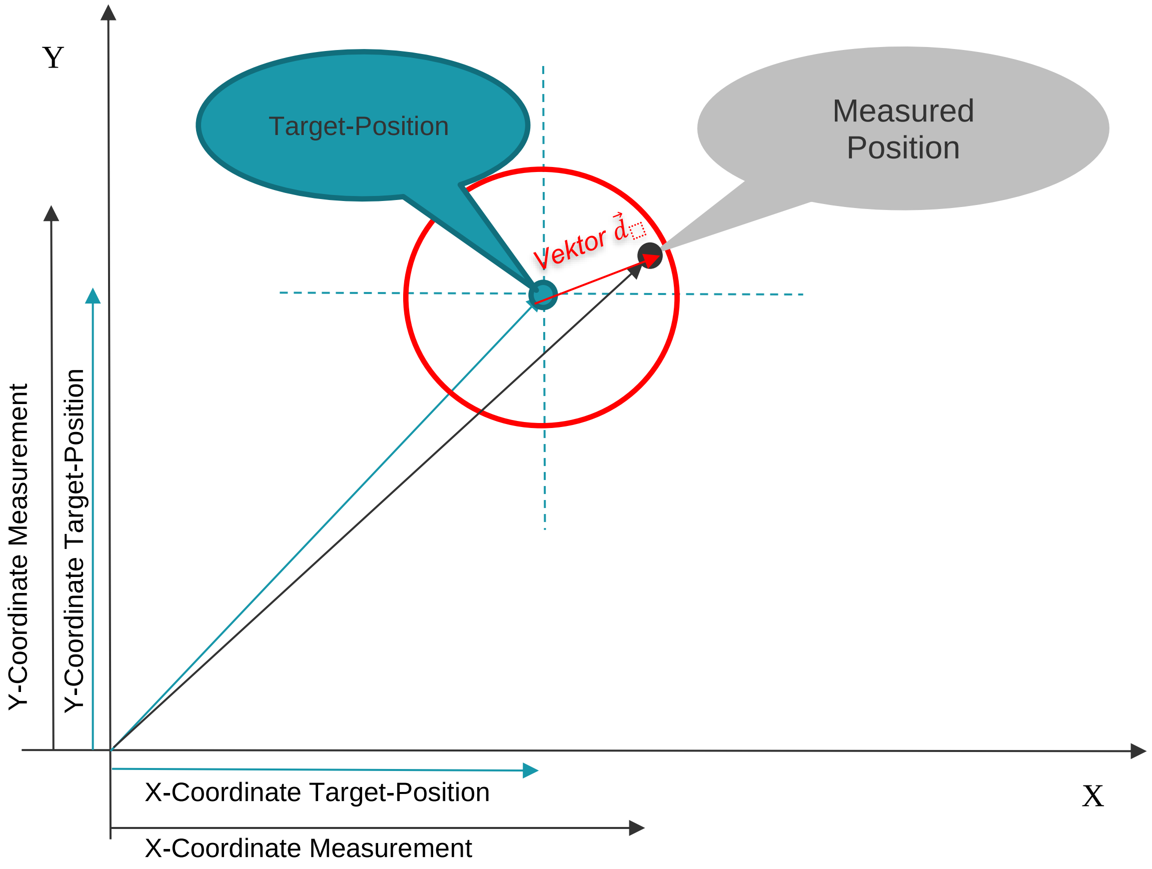


Figure 1-4: Sketch not drawn to scale to illustrate the two position vectors for the actual and target position and the Variation of spread vector d for the position deviation

1.3 Position deviation as difference of the position vectors

We obtain the vector of the position deviation from the difference of the two location vectors for the current and target position:

$$\vec{P}_{curr} - \vec{P}_{tar} = \begin{pmatrix} \Delta x = x_{curr} - x_{tar} = 30{,}00\ mm \\ \Delta y = y_{curr} - y_{tar} = 20{,}00\ mm \end{pmatrix} = \begin{pmatrix} 30{,}05mm - 30{,}00\ mm \\ 20{,}04mm - 20{,}00\ mm \end{pmatrix} = \begin{pmatrix} 0{,}05\ mm \\ 0{,}04\ mm \end{pmatrix}$$

The shortest distance between actual and target position corresponds to the length of this vector.

1.3.1 Determining the length of the difference vector (actual position deviation)

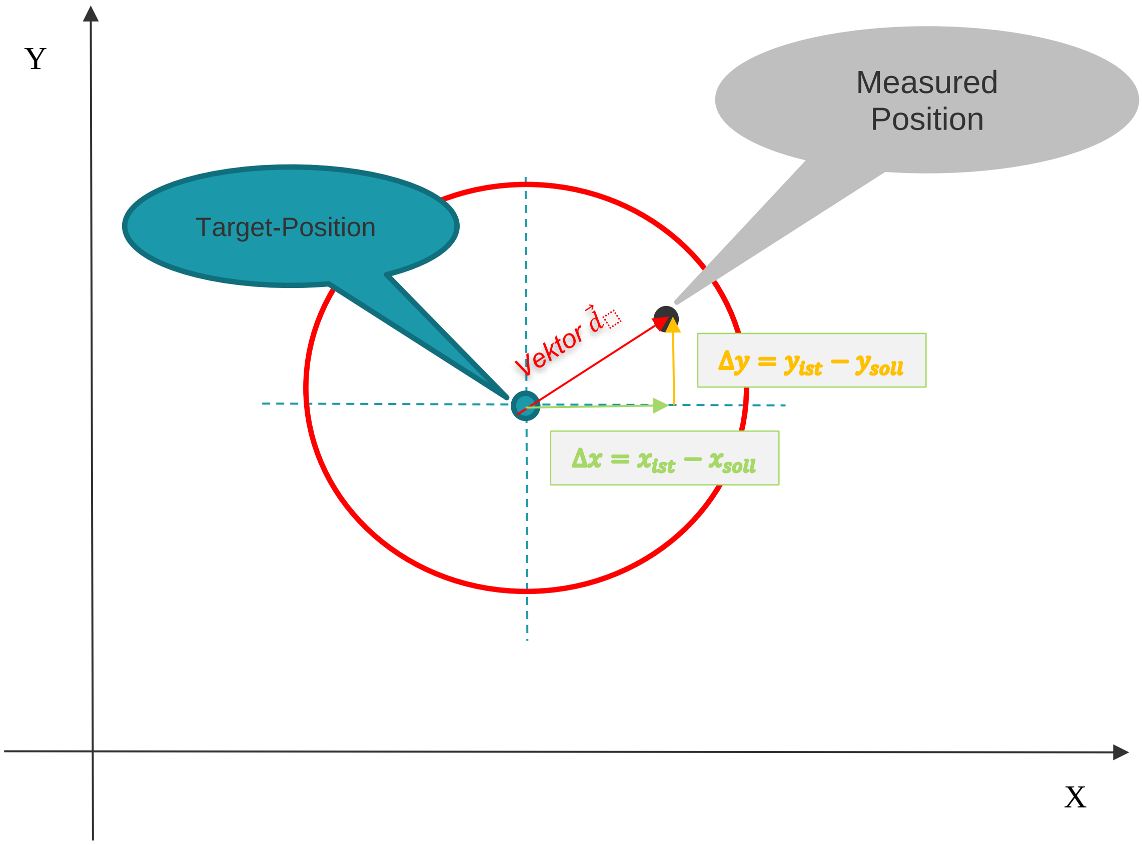


Figure 1-5: Sketch not drawn to scale to illustrate the Variations vector d, which describes the deviations of the actual position from the nominal position.

The magnitude or length of the vector $\vec{d}$ is the Euclidean distance between the actual and the nominal position:
$$|\vec{d}| = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_{ist} - x_{soll})^2 + (y_{ist} - y_{soll})^2}$$

If we use the Values from our numerical example, we get:
$$|\vec{d}| = \sqrt{(0{,}05^2 + 0{,}04^2)mm^2} \approx 0{,}06403mm$$

As a rule, we do not use the simple amount $|\vec{d}|$ of the vector, as this only expresses the radial distance between the actual and target position. The Positional tolerances t_{PS} is given to us as a diameter. Therefore, it is obvious and also common to output the observed position deviation f_{PS} as a diameter as well:

$$f_{PS} = 2 \cdot |\vec{d}| \approx 2 \cdot 0{,}06403mm = 0{,}12806mm$$

Note: We can set whether the calculated position deviations are to be output as radius or diameter in the evaluation strategy of the software (administrator rights required).

To view or change the currently active calculation option, we select the menu command:
Start | Evaluation strategy

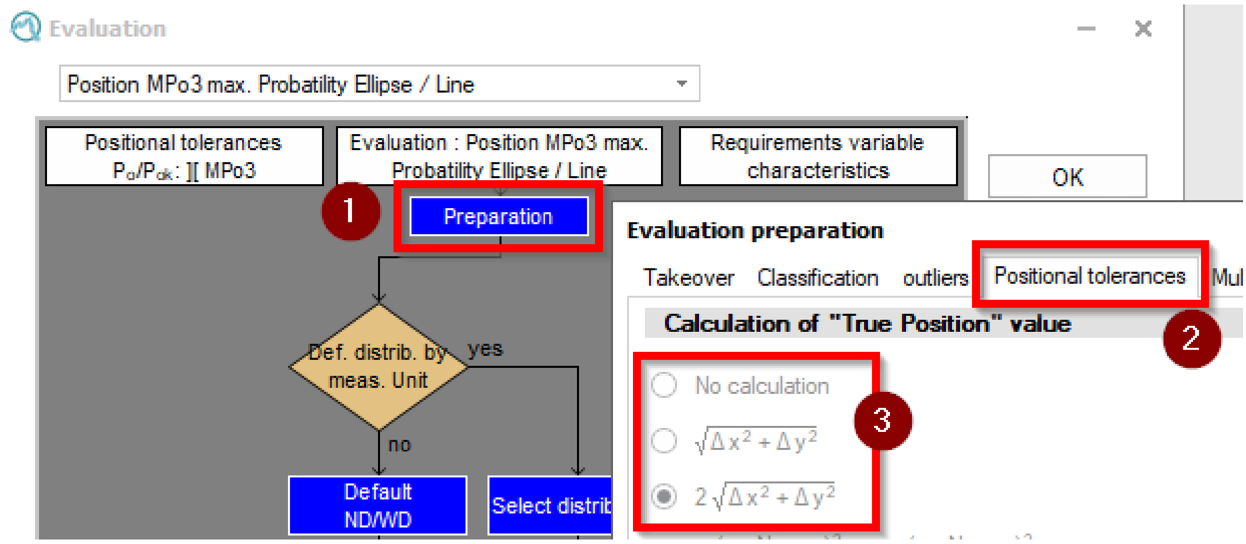


Figure 1-6: Calculation options for the "true-position" value in the software

If the setting "no calculation" is active, no position deviation amounts are calculated and therefore not output.

1.3.2 Checking the Requirement for the Position Deviation

With the "deviation diameter" f_{ps} from section 1.3.1 we check whether the Requirement for the position deviation is fulfilled. The acceptance condition is in words: The observed Actual Value for the position deviation f_{ps} should be less than or at most equal to the value of the position deviation tolerance t_{PS}. This can be expressed "succinctly" as a formula.

$f_{PS} \leq t_{PS}$

If the condition is fulfilled, the currently measured position is "OK".
However, the assessment of individual units only makes sense if really every unit can be tested and assessed according to the criterion mentioned. Such 100% testing of the units is often not feasible due to the excessive duration of test and cost.

One way out is monitoring with subgroups: We take a random subgroups with e.g. n = 5 units at regular intervals or after a fixed number of units and use this subgroup to check whether the process has manufactured the *positional deviations* f_{ps} "process-safely" within the *positional tolerances* t_{ps}. However, the use of statistical monitoring with subgroups is tied to the application prerequisite that the manufacturing process is able to produce the positions "process-safe”

2 Evaluate position deviations with qs-STAT

In the first step, we create a position characteristic by hand in the Sample Analysis module of the qs-STAT programme.

Note: Many producers of measuring machines equip their measuring machines with an interface for the Q-DAS ASCII transfer format, so that we users do not have to worry about creating and entering manually.

We start the programme qs-STAT and select *Start* | *Module selection* | *Sample analysis*

Now we create a new Characteristic for the position deviation. We select *File* | *New*

The window "Create new characteristics..." appears.

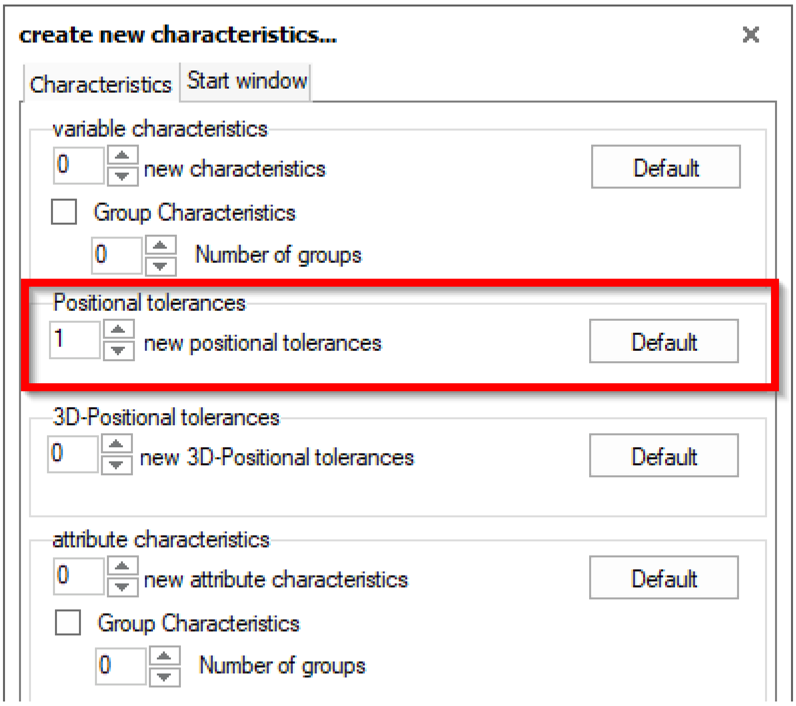


Figure 2-1: Window "Create new characteristics" with the activated option "1 new Positional tolerances".

In the window we set the Positional tolerances option to the value 1 and confirm our selection with OK. Now the following programme view appears:

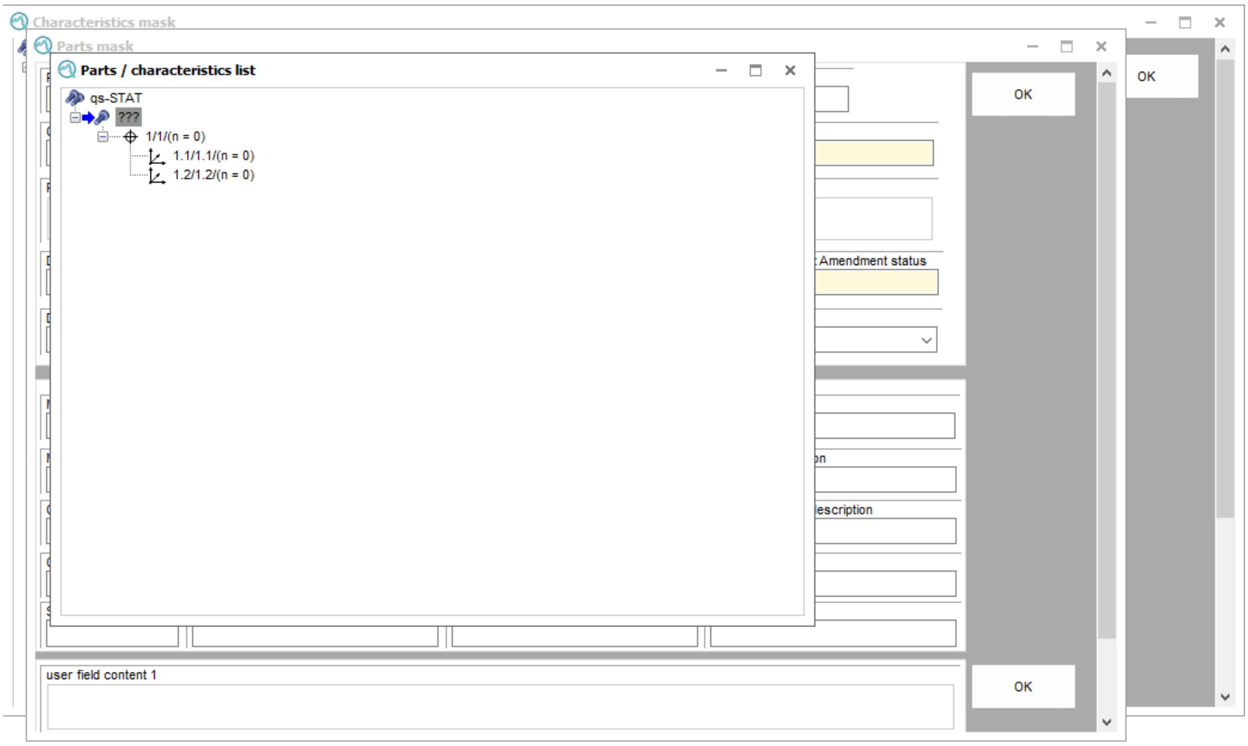


Figure 2-2: View of the programme after creating a new Positional tolerance

As can be seen from Figure 2-2 in the Parts / Characteristics List window, the Positional tolerances is a Characteristics group, which includes three Characteristics:

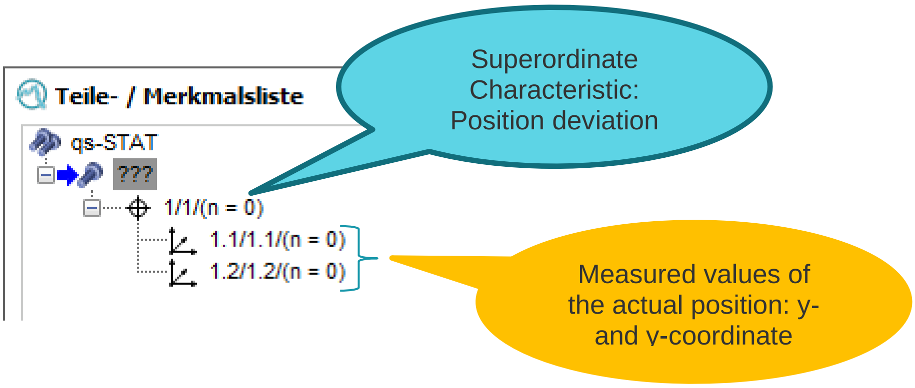


Figure 2-3: Characteristics group Positional tolerances - consisting of the superordinate characteristic Positional deviation and the two subordinate characteristics for position measurement values per coordinate

Now we click on the "Parts mask" window (if this was closed, we click *Start* | *Parts* mask) and fill it in as follows:

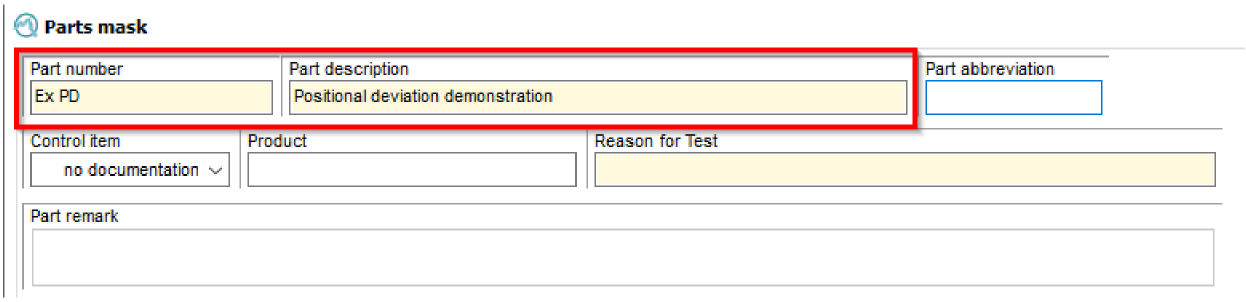


Figure 2-4: Window "Parts mask", in which we only enter the part number "Ex PD" and the part designation "Positional deviation demonstration".

We close the "Parts mask" window by clicking on OK. We now see the "Characteristics mask" window. If we have already closed this window by mistake, we open it again with the command Start | Characteristics mask.

To make it easier to see for which of the three characteristics we are editing the fields in the characteristics mask, we show the characteristics list. To do this, we click in the menu on:

Graphic settings | "Info" symbol

The window "Info" appears, in which we activate the option characteristic list. We confirm the setting with OK.

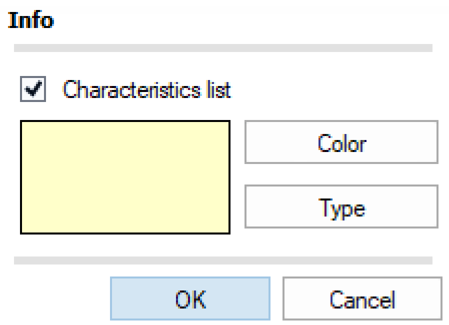


Figure 2-5: Info window with activated feature list

By activating it, we see the characteristics list at the left edge of the "Characteristics mask" window, in which the characteristics are displayed with the already known group structure.

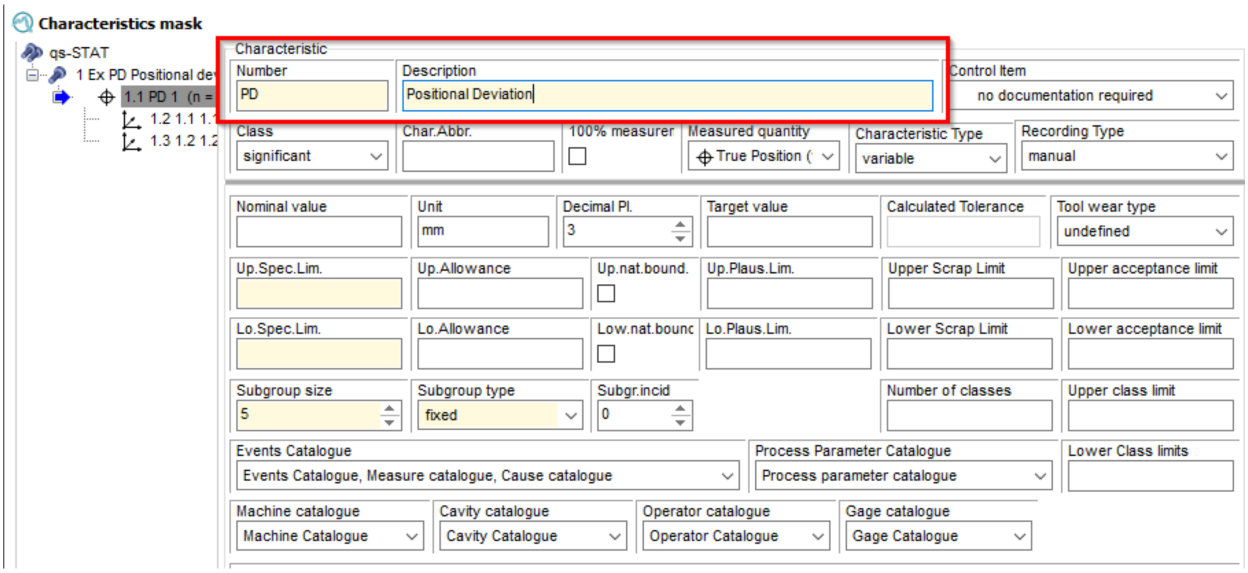


Figure 2-6: View of the "Characteristics mask" window with the "Characteristics list" option activated

We click on the parent Characteristic for the position deviation in the Characteristics list, which gives us access to the Properties fields for that Characteristic. In these we enter:

Field name	Input value	remark
Number	PD	
Description	Positional Deviation	
Unit	mm	Entry already exists
Measured quantity	True Position (Betrag)	Entry already exists

Now we switch to the first subordinate characteristic in the list of characteristics with a mouse click:

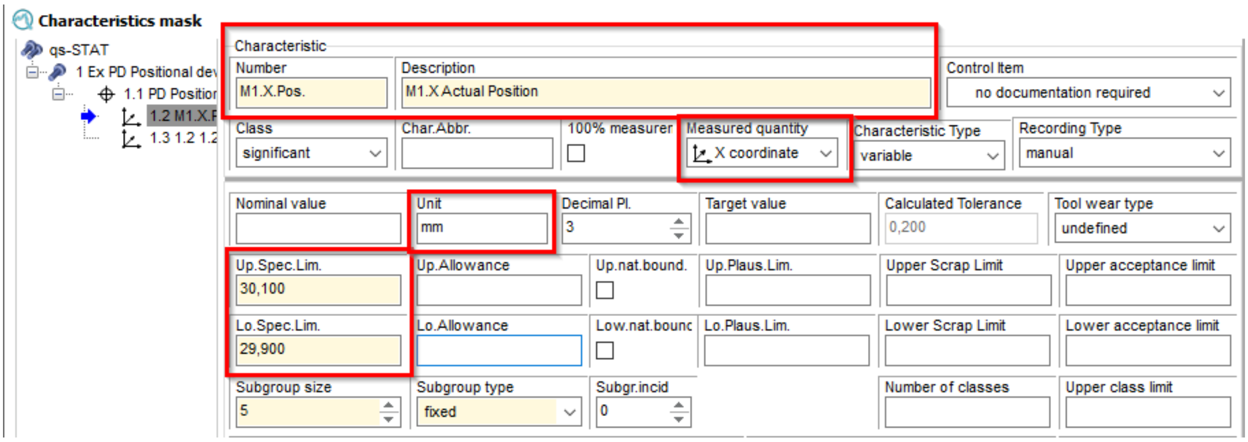


Figure 2-7: View of the Characteristics mask window for the first subordinate characteristic (entry already made)

We enter here:

Field name	Input value	remark
Number	M1.X.Pos.	
Description	M1.X Actual Position	
Up. Spec. Lim.	30,1	
Low. Spec. Lim.	29,9	
Unit	mm	Entry already exists
Measured quantity	X-Koordinate	

With reference to the example sketch in Figure 1-1 on page 3, we know that the *nominal position* of the borehole has the point coordinates (X_{tar} = 30 mm | Y_{tar} = 20 mm) and the associated Positional tolerances are tPS = 0.2 mm. In the programme we have to calculate this Positional tolerances according to the following scheme

$$Target \pm t_{PS}/2$$

for each of the coordinate axes.

The following Specification limits therefore result for the X-coordinate:

$$USL = X_{tar} + \frac{t_{ps}}{2} = 30mm + \frac{0{,}2mm}{2} = 30{,}1mm$$

$$LSL = X_{tar} - \frac{t_{ps}}{2} = 30mm - \frac{0{,}2mm}{2} = 29{,}9mm$$

We switch to the second subordinate characteristic and fill in the associated fields:

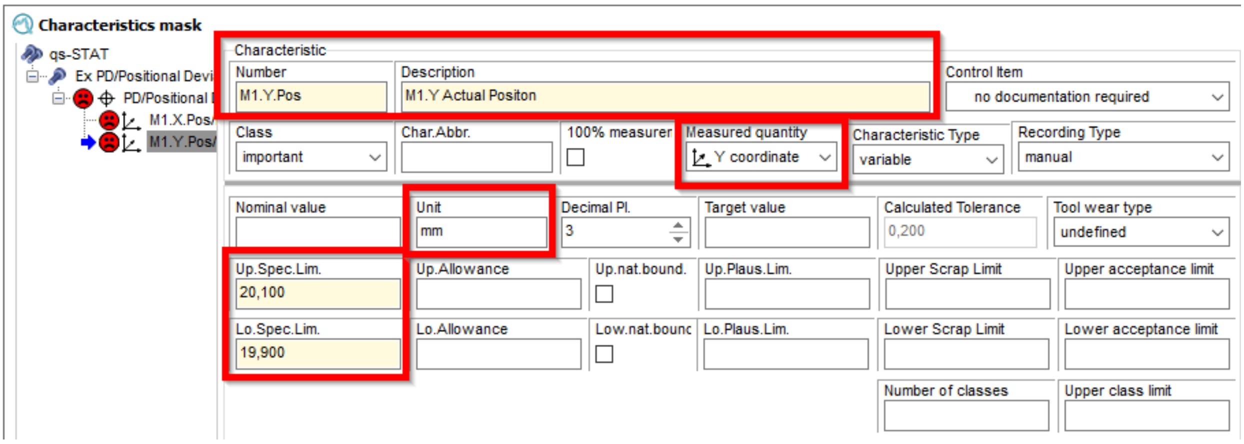


Figure 2-8: View of the Characteristics Mask window after selecting the third characteristic (values already entered))

Field name	Input value	remark
Number	M1.Y.Pos.	
Description	M1.Y Actual Position	
Up. Spec. Lim.	20,1	
Low. Spec. Lim.	19,9	
Unit	mm	Entry already exists
Measured quantity	X-Koordinate	

With reference to the example sketch in Figure 1-1 on page 3, we know that the *nominal position* of the borehole has the point coordinates ($X_{tar} = 30\ mm$ | $Y_{tar} = 20\ mm$) and the associated Positional tolerances are $tPS = 0.2\ mm$. In the programme we have to calculate this Positional tolerances according to the following scheme

$$Target \pm t_{PS}/2$$

for each of the coordinate axes.

The following Specification limits therefore result for the X-coordinate:

$$USL = Y_{tar} + \frac{t_{ps}}{2} = 20mm + \frac{0{,}2mm}{2} = 20{,}1mm$$

$$LSL = Y_{tar} - \frac{t_{ps}}{2} = 20mm - \frac{0{,}2mm}{2} = 19{,}9mm$$

The programme calculates the tolerance for the superordinate characteristic "position deviation" from the specification limits entered for the coordinate axes. I.e. **we do not need to enter specification limits for the superordinate characteristic "position deviation"**.

Our next step is to enter the position measurement results in the "Value mask" window, which we call up as follows:

Start | Value mask

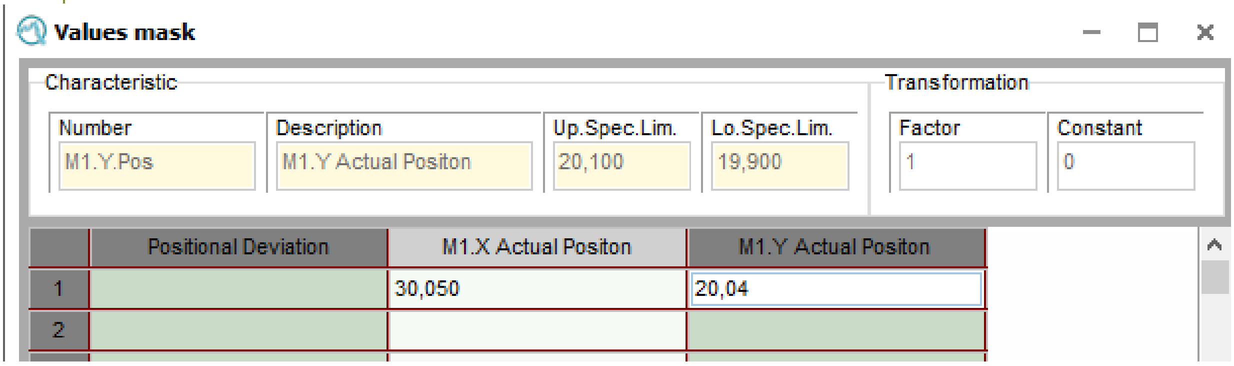


Figure 2-9: View of the Values mask window with the first entered pair of values of the measured actual positions of the borehole (X- and Y-coordinate))

We leave the first column with the designation "Positional Deviation" empty. This is, after all, the higher-level characteristic of the positional deviation. If the calculation option for the positional deviation amount is activated, the programme calculates the positional deviation amount automatically as soon as we have entered the positional measurement results.

M1.X Actual Postion = *30,05 mm* und M1.Y Actual Postion = *20,04 mm*

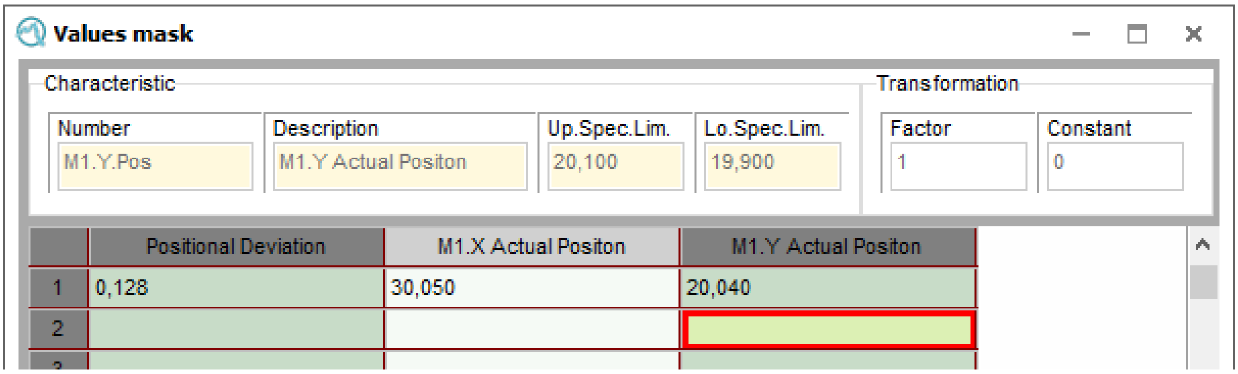


Figure 2-10: View of the Values mask after confirming the second input value - The higher-level Characteristic Position Deviation is calculated automatically[1]

[1] If the calculation is activated in the evaluation strategy, which is the case in the evaluation strategy used here "Q-DAS Machine Capability (06/2013)" is the case.

Now we have seen how the creation and input basically works. We prefer to dispense with the manual entry of further measurement results here and instead load an appropriately prepared file with 50 pairs of values.:

File | File open

In the file dialogue window we select the file "Positional_Deviation_Example.dfq" and confirm our selection with the "Open" command.

2.1 Numerics evaluation

What confuses many users when they first come into contact with Positional tolerances is the fact that we do not use the calculated Positional tolerances in our "qs-STAT" and "destra" programmes for the process capability evaluation. In principle, the evaluation of the deviation amounts can be set in the evaluation strategy, but this has a decisive disadvantage:

We lose the information about the two-dimensional scattering behaviour in the evaluation of the deviation amount. For this reason, we use the calculation method "MPo2 max. Probability ellipse" in our standard evaluation strategies[2] in the modules "Sample analysis" and "Process analysis", in which the two-dimensional scattering behaviour is taken into account.

For the evaluation we choose:

Results | form sheets

The window "Form 3" opens, which contains the evaluation results for the superior characteristic (position deviation).

[2] In the module "Sample Analysis" the standard evaluation strategy is "Q-DAS Machine Capability (06/2013)" and in the module "Process Analysis" the standard evaluation strategy is "Q-DAS Process Capability (06/2013)".

Part no.	Ex PD	Part descr.	Positional Deviation Example		
Char.No.	PD	Char.Descr.	Positional Deviation		
Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07632
T*	0.200	R	0.200	$X_{0.135\%}$	-0.00081 [rt]
Characteristics C:	unimportant	n_{eff}	50	$X_{99.865\%}$	0.23057 [rt]
		n_{tot}	50	$X_{up3}-X_{lo3}$	0.23138 [rt]
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.32245 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.67755%
		$n_{<LSL}$	---	$p_{<LSL}$	---
Model distribution				Weibull distribution	
Distr.regress.coeff.		r_{tot}	:	0.99559428	
Distr.regress.coeff.		$r_{25\%}$	:	0.97230551	
Calculation method			MPo2 max. probability ellipse		
Potential Capability index		P_o	0.68 ≤ **0.84** ≤ 1.01	0 1.67	
Critical capability index		P_{ok}	0.55 ≤ **0.72** ≤ 0.88	0 1.67	
The requirements were not met (P_o,P_{ok})					
Demand Potential Capability index		$P_{o\ target}$	1.67		
Demand Critical capability index		$P_{ok\ target}$	1.67		
Q-DAS Machine Capability (06/2013)					

Figure 2-11: Window "Form - Display 3" with the evaluation results for the superordinate Characteristic Position Deviation

The details of the characteristic value determination according to the calculation method "MPo2 max. Probability ellipse" can be found in section 3.4 on page 29.

On the basis of Figure 2-11 we can see:

The ***capability index*** $\boldsymbol{P_o} = 0.84$ is smaller than the *default value* $P_{ok_{min}} = 1.67$, therefore the **Requirement not fulfilled**. The reason is the too large deviation of the positions.

The ***minimum capability index*** $\boldsymbol{P_{ok}} = 0.72$ is smaller than the target *value* ${}_{Pokmin} = 1.67$, so this **requirement** is also **not met**. In addition to the scatter being too large, the mean position of the position measurement values is also shifted from the nominal position.

Overall, the deviation of the borehole positions is too large. We cannot generate the borehole positions "safely" within the position tolerance.

2.2 Graphical evaluation

We select the menu command:

Graphics | Position Tolerances | X-Y Plot Position

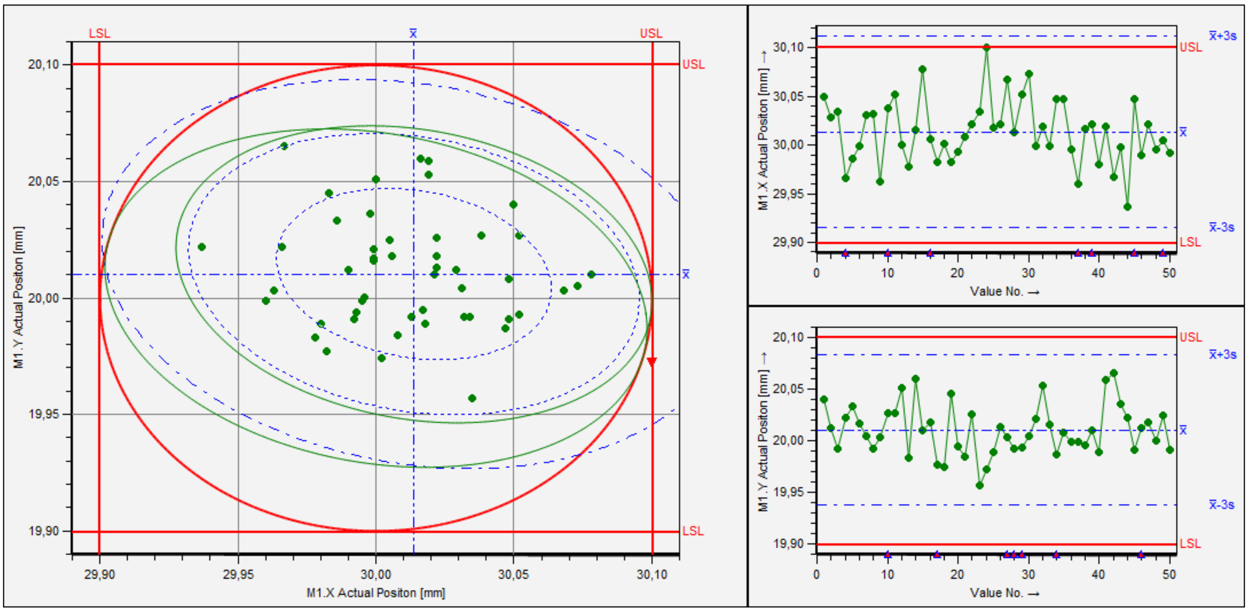


Figure 2-12: View of the x-y plot position window with the values of the case study.

In Figure 2-12, the *tolerance circle* with diameter t_{PS} = 0.2 mm is shown in red line color. The large green scatter ellipse belongs to the characteristic value P_o and the small green scatter ellipse belongs to the characteristic value P_{ok}. For individual details of the capability calculation, see Section 3.4.

We can obtain further graphs for position tolerances with the commands listed below. These graphs are intended for data sets with several position features, when these are to be compared with each other:

Graphics | Position Tolerances | Capability Indices Graphics |

Box Plot Position

3 Types of capability calculation for position deviations

In this chapter we look at the four calculation types available in the software for the capability evaluation of position deviations. All calculation type settings described in this chapter refer to the overall evaluation of the position deviation characteristic group. For some evaluations, the evaluation strategy had to be adapted. The sections affected by this contain corresponding notes.

Note: The **presentation of the calculation steps** is **not** 100% **identical with the algorithms** as implemented **in the programs qs-STAT, procella and destra. The algorithms** implemented in our programs are therefore not presented here.

All calculation steps presented here are intended to help the reader understand how to get from the position measurement results to the individual parameters of the process performance and capability. This understanding is necessary in order to be able to assess the statement and significance of the individual parameters and to make decisions for settings in the evaluation strategy.

3.1 Setting options for the calculation type (qs-STAT/destra)

We call up the evaluation strategy view via the ribbon:

Start | Evaluation strategy

The "Evaluation" window appears, which contains a flowchart graphic of the evaluation steps. In the upper left corner of the flow chart graphic, the white rectangle with the label "Position tolerances Po / Pok: MPo2" can be seen.

Note: If a different calculation type is set in the evaluation strategy used by the reader, the labeling of the box is usually also different than shown here.

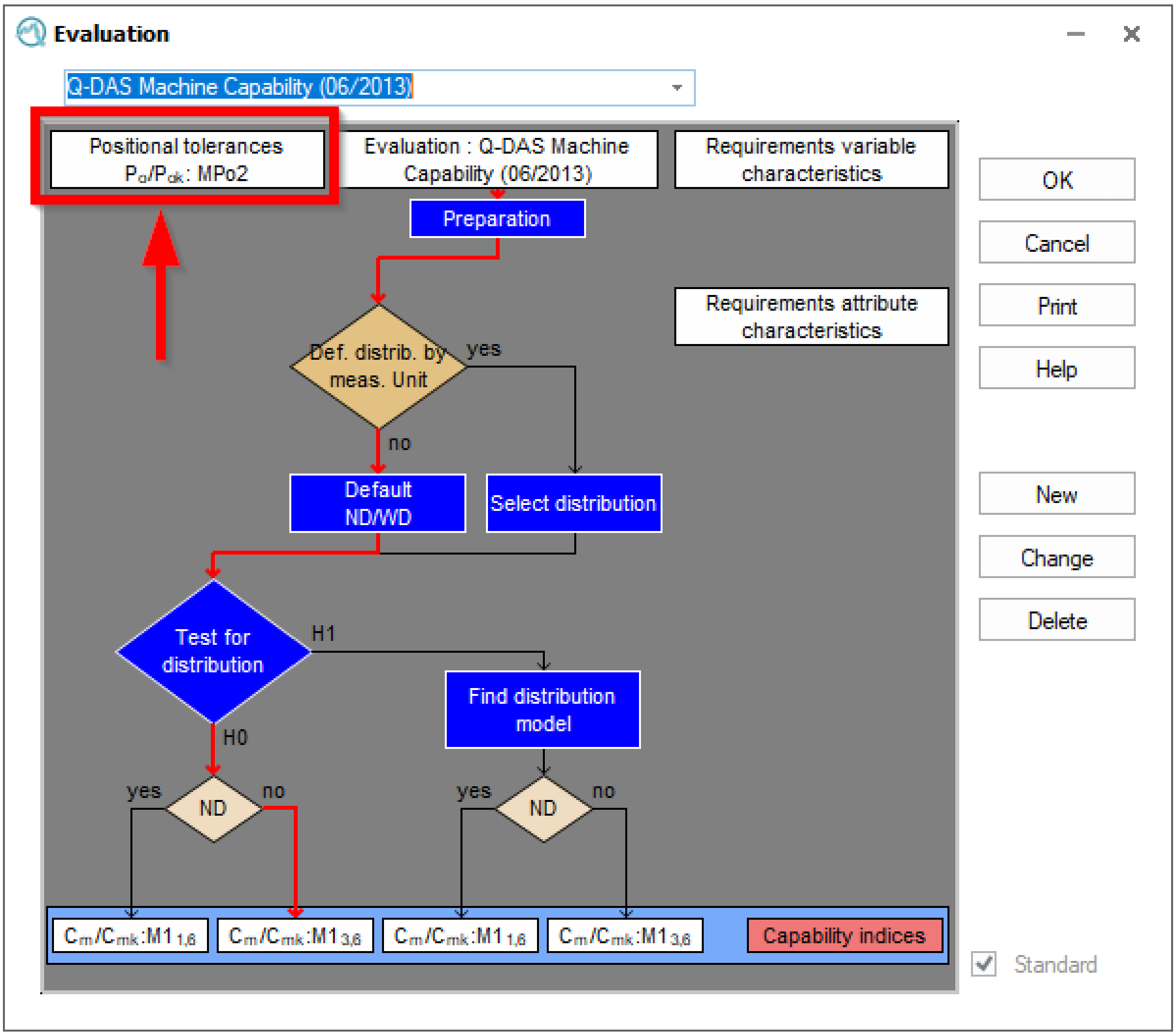


Figure 3-1: Evaluation window containing the evaluation strategy as a flowchart

If we click with the mouse on this box, the window "Requirements position tolerances" opens.

In the window "Requirements position tolerances" we click on the tab "Calculation type".

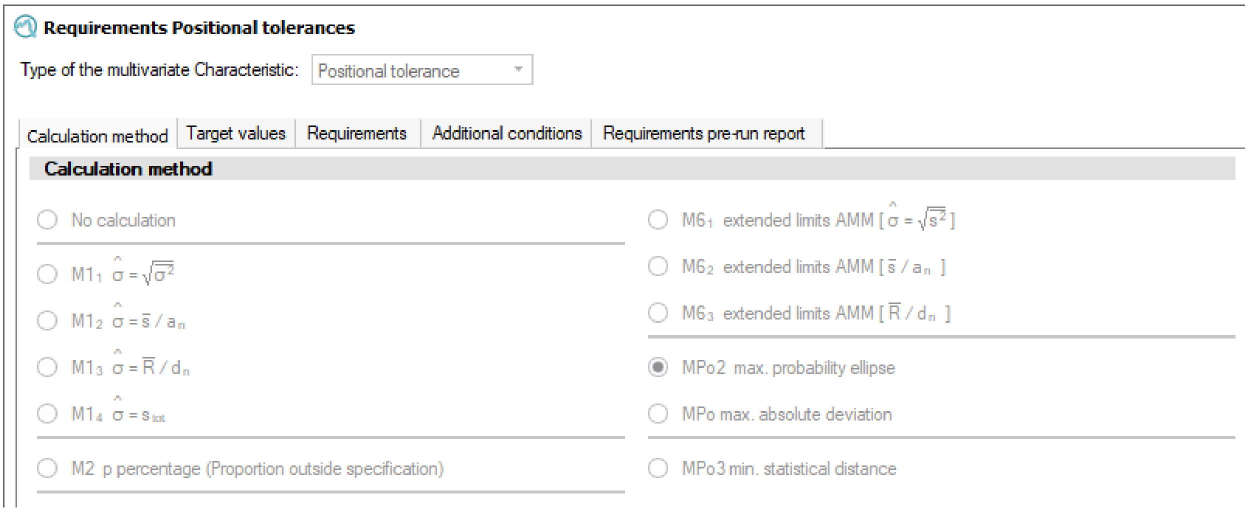


Figure 3-2: "Position Tolerance Requirements" window - "Calculation Type" tab

Figure 3-2 shows the corresponding excerpt from the "Q-DAS Machine Capability (06/2013)" strategy. This evaluation strategy is the default setting, provided that no company-specific adjustments have been made (as of spring 2016). The calculation type activated in it is the procedure "MPo2 max. probability ellipse", which corresponds to the procedure "Type I" from the *ISO 22514-6:2013* standard.

3.2 One-dimensional evaluation of the “true position” value

We should always evaluate the two-dimensional, circularly defined position tolerances in two dimensions in order to have taken into account the two-dimensional scattering behaviour of the values. **The one-dimensional evaluation method described here first is not recommended for use,** since this type of evaluation obscures the two-dimensional scattering behavior of the position measurement results.

To activate a one-dimensional evaluation for the superordinate characteristic "Positional Deviations", we create a new evaluation strategy (administrator rights required!) and activate one of the available calculation types for one-dimensional characteristics in it. In the following figure, the calculation type " $M4_2$ Percentil (0,135 %-50 % - 99,865 %)" has been selected as an example, which corresponds to the calculation method $M_{l=2,\ m=1}$ in the standard ISO 22514-2:2013

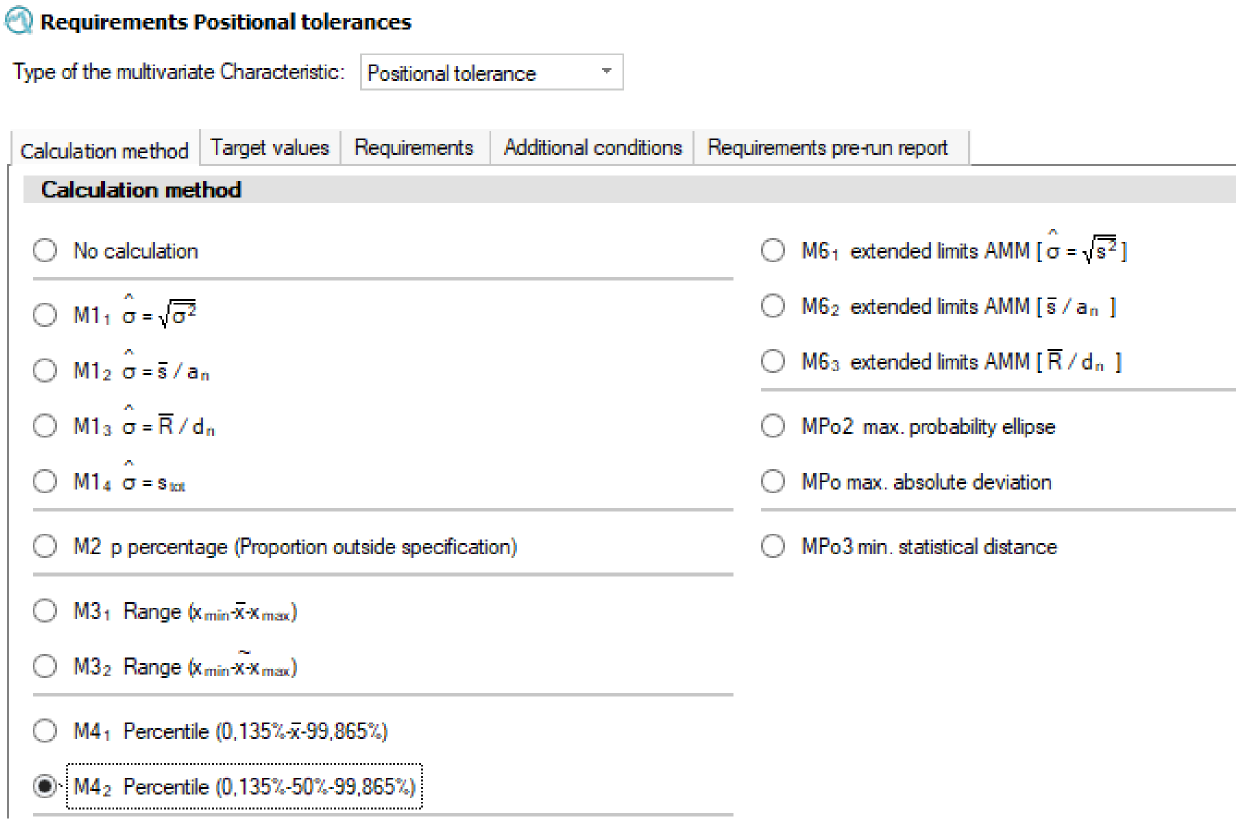


Figure 3-3: Setting for the one-dimensional evaluation of the deviation amount (for calling up the window, see section 3.1 on page 21)

Using this method, we obtain the evaluation results shown in Figure 3-5 for the case study data.

Note: Please note that for two-dimensional, circularly defined position tolerances, the one-dimensional evaluation obscures the two-dimensional scattering behavior.

To the values of the characteristic "Positional Deviation" from the sample data set "Positional_Deviation_Example.dfq", the program selected and fitted the model "Weibull distribution" as the best fitting model. Furthermore, the program determined the two scatter limits of the 99.73 % random scatter range of this distribution:

- Lower variation limit = 0,135 %-Quantil $Q_{0,135\,\%}$of the Weibull-distribution
- Upper variation limit = 99,865 %-Quantil $Q_{99,865\,\%}$ of the Weibull-distribution

The calculated values of these quantils are:

$$Q_{0,135\,\%} = -0{,}000\,81\,mm$$

$$Q_{99,865\,\%} = 0{,}230\,57\,mm$$

In the histogram (see: Figure 3-4), the 0.135% quantile of the Weibull distribution is shown as line Q_{un3} and the 99.865% quantile of the Weibull distribution is shown as line Q_{ob3}.

Using these quantile values, we determine the process performance indices as follows:

$$P_o = \frac{OSG - USG}{Q_{99,865\%} - Q_{0,135\%}} = \frac{(0{,}2 - 0)mm}{[0{,}23057 - (-0{,}00081)]} = \frac{0{,}2mm}{0{,}23138} \approx 0{,}86$$

$$P_{ok} = \frac{OSG - Q_{50\%}}{Q_{99,865\%} - Q_{50\%}} = \frac{(0{,}2 - 0{,}07211)mm}{[0{,}23057 - 0{,}07211]} \approx 0{,}81$$

Note: The lower specification limit LSL is a natural limit and for this reason is ignored in the P_{ok} calculation

We obtain the histogram of the position deviation amounts by opening the "Histogram - Single Values" window with the function key F4 or with the command Graphics | Histogram.

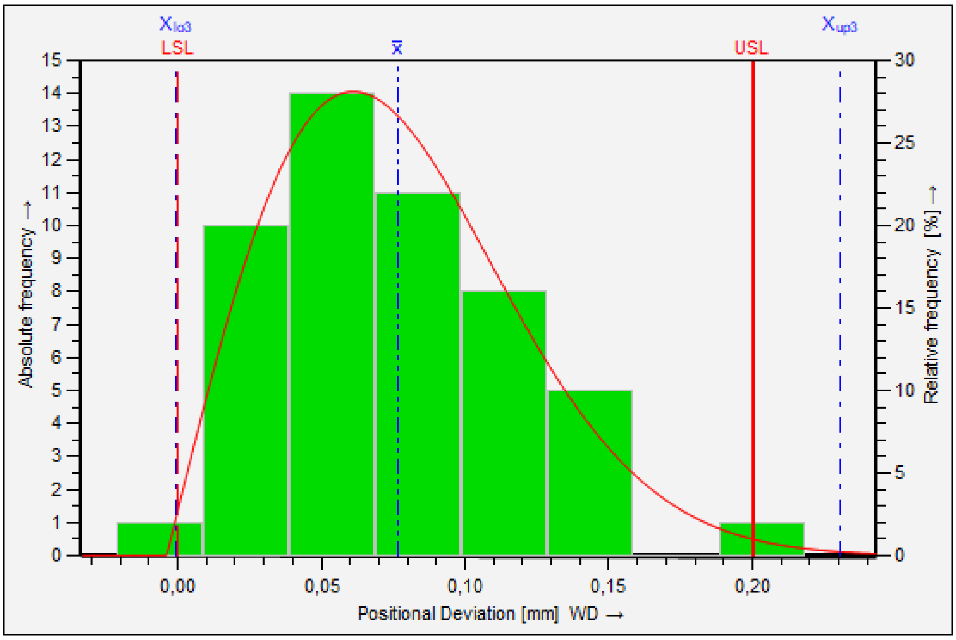


Figure 3-4: Graphic histogram for the characteristic "Positional Deviation" with the distribution model Weibull distribution fitted to it (Graphics | Histogram)

We open the results window "Form - Representation 3" by pressing the function key F10. Alternatively, we can select the command Results | Form Sheets.

The following result was generated with the program "qs-STAT" in the module "Sample Analysis" on the basis of the evaluation strategy " Po Pok univariat Absolut Value "[3], which cannot be selected in the program.

Part no.	Ex PD	Part descr.	Positional Deviation Example
Char.No.	PD	Char.Descr.	Positional Deviation

Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07632
T*	0.200	R	0.200	$X_{0.135\%}$	-0.00081 [rt]
Characteristics C :	unimportant	n_{eff}	50	$X_{99.865\%}$	0.23057 [rt]
		n_{tot}	50	X_{up3}-X_{lo3}	0.23138 [rt]
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.32245 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.67755%
		$n_{<LSL}$	---	$p_{<LSL}$	---

Model distribution			Weibull distribution
Distr.regress.coeff.	r_{tot}	:	0.99559428
Distr.regress.coeff.	$r_{25\%}$	:	0.97230551
Calculation method		$M1_{3,6}$	Percentile (0,135%-50%-99,865%)
Potential Capability index	P_p	0.69 ≤ **0.86** ≤ 1.03	0 … 1.67
Critical capability index	P_{pk}	0.62 ≤ **0.81** ≤ 0.99	0 … 1.67
The requirements were not met (P_p,P_{pk})			
Demand Potential Capability index	$P_{p\ target}$	1.67	
Demand Critical capability index	$P_{pk\ target}$	1.67	
Po Pok univariat Absolut Value			

Figure 3-5: Window "Form - Display 3" with the evaluation result according to the univariate evaluation M2,1 according to ISO 22514-2:2013 for the example data set "positional_deviation_example.dfq" (call with function key F10)

[3] The evaluation strategy " Po Pok univariat Absolut Value " was created temporarily for the calculation demonstration and was not included in the list of evaluation strategies available in the program due to its minor importance.

3.3 Calculation type "MPo max. absolut deviation

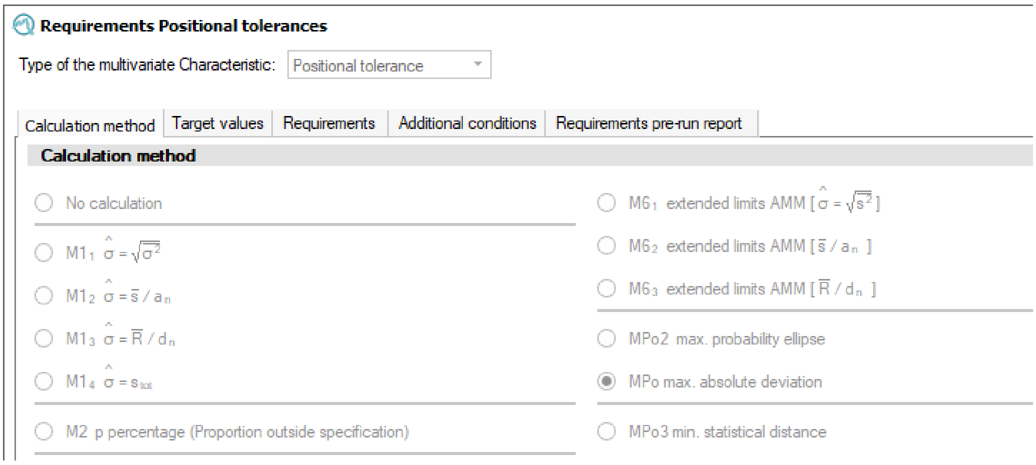


Figure 3-6: "Position tolerance requirements" window with the set calculation type "MPo max. absolut deviation " (to call up this setting window, see section 3.1 on page 21)

In the first step, we look at the x-y-plot of the position measurement results and pick the value that has the largest radial distance $d_{euk.max}$ from the nominal position

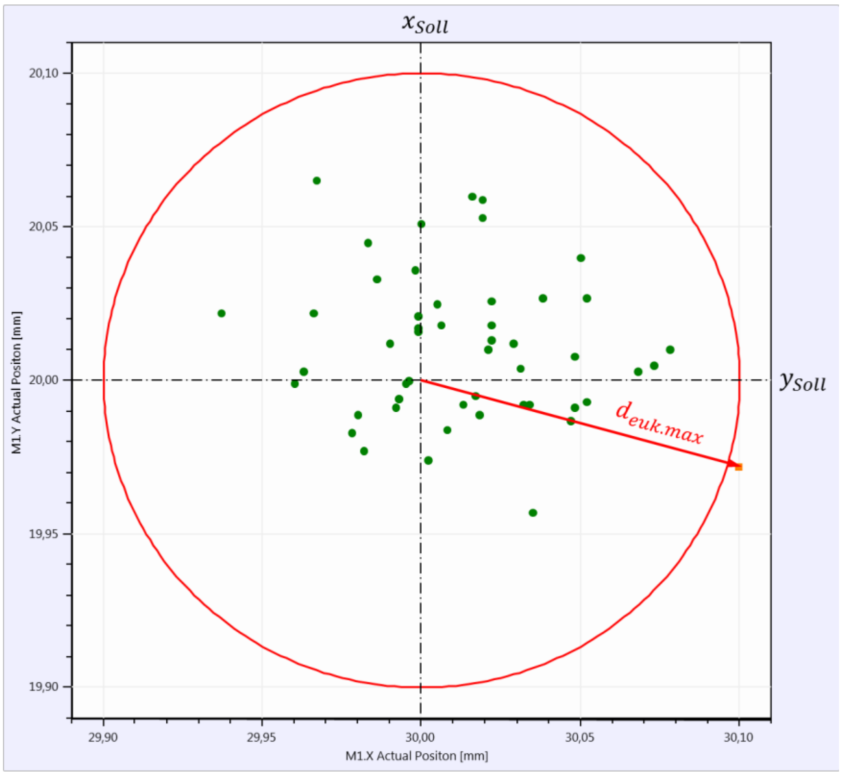


Figure 3-7: Representation of the position measurement results in the x-y-plot with highlighting of the measurement result that has the largest radially measured distance from the nominal value

Formally, the largest radial distance corresponds to the largest value of all Euclidean distances, calculated from the actual and nominal positions:

$$d_i = \sqrt{(x_i - x_{tar})^2 + (y_i - y_{tar})^2}; i = 1,2,\ldots,n$$

$$d_{euk.max} = \max(d_1, d_2, \ldots, d_n)$$

With:

x_i= x -coordinate of the actual position ; y_i = y -coordinate of the actual position.

x_{tar} = x -coordiante of nominal position ; y_{tar} = y -coordiante of nominal position

n = number of values

The value with the greatest radial distance from the nominal position has the actual position:

$$P_{\mathrm{curr}} = (x_{i=24} = 30{,}100\ mm \mid y_{i=24} = 19{,}972\ mm)$$

As we can see from Figure 1-1 on page 3, the coordinates for the target position are:

$$P_{\mathrm{tar}} = (x_{\mathrm{tar}} = 30{,}000\ mm \mid y_{\mathrm{tar}} = 20{,}000\ mm)$$

With this information we calculate the maximum Euclidean distance:

$$d_{euk.max} = \sqrt{(30{,}100 - 30{,}000)^2 mm^2 + (19{,}972 - 20{,}000)^2 mm^2} \approx 0{,}10385\ mm$$

In the second step, we divide the tolerance circle radius ($= t_{PS}/2$) by the Euclidean distance. The result is the *minimum capability index* P_{ok}:

$$P_{OK} = \frac{\left(\frac{t_{ps}}{2}\right)}{d_{euk.max}} \approx \frac{0{,}1mm}{0{,}10385mm} \approx 0{,}96$$

The following result was generated with the program "qs-STAT" in the module "Sample analysis" on the basis of the evaluation strategy "Po Pok univariat Position Absolut Deviation"[4], which cannot be selected in the program.

Part no.	Ex PD	Part descr.	Positional Deviation Example
Char.No.	PD	Char.Descr.	Positional Deviation

Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07632
T*	0.200	R	0.200	$X_{0.135\%}$	-0.00081 [rt]
Characteristics C :	unimportant	n_{eff}	50	$X_{99.865\%}$	0.23057 [rt]
		n_{tot}	50	X_{up3}-X_{lo3}	0.23138 [rt]
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.32245 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.67755%
		$n_{<LSL}$	---	$p_{<LSL}$	---

Model distribution			Weibull distribution
Distr.regress.coeff.	r_{tot}	:	0.99559428
Distr.regress.coeff.	$r_{25\%}$	:	0.97230551
Calculation method			MPo max. absolute deviation
Potential Capability index	P_o	--- 989	0 1.67
Critical capability index	P_{ok}	0.96	0 1.67
--- 11	Capability indices were not calculated		--- 11
Demand Potential Capability index	$P_{o\ target}$	1.67	
Demand Critical capability index	$P_{ok\ target}$	1.67	
Po Pok univariat Position Absolut Deviation			

Figure 3-8: Window "Form - Display 3" with the evaluation result after the calculation type "MPo max. deviation amount" for the example data set "positional_deviation_example.dfq" (call with the function key F10)

Note: We have to keep in mind that this type of calculation **uses very little information from the sample data.** Only one extreme value (!) is used for the ability calculation, which is why we do not recommend using this type of calculation.

[4] The evaluation strategy "Po Pok univariat Position Absolut Deviation" was created temporarily for the calculation demonstration and was not included in the list of evaluation strategies available in the program due to its minor importance.

3.4 Calculation type "MPo2 max. probability ellipse“

The program fits a two-dimensional normal distribution to the two-dimensional measured actual positions.

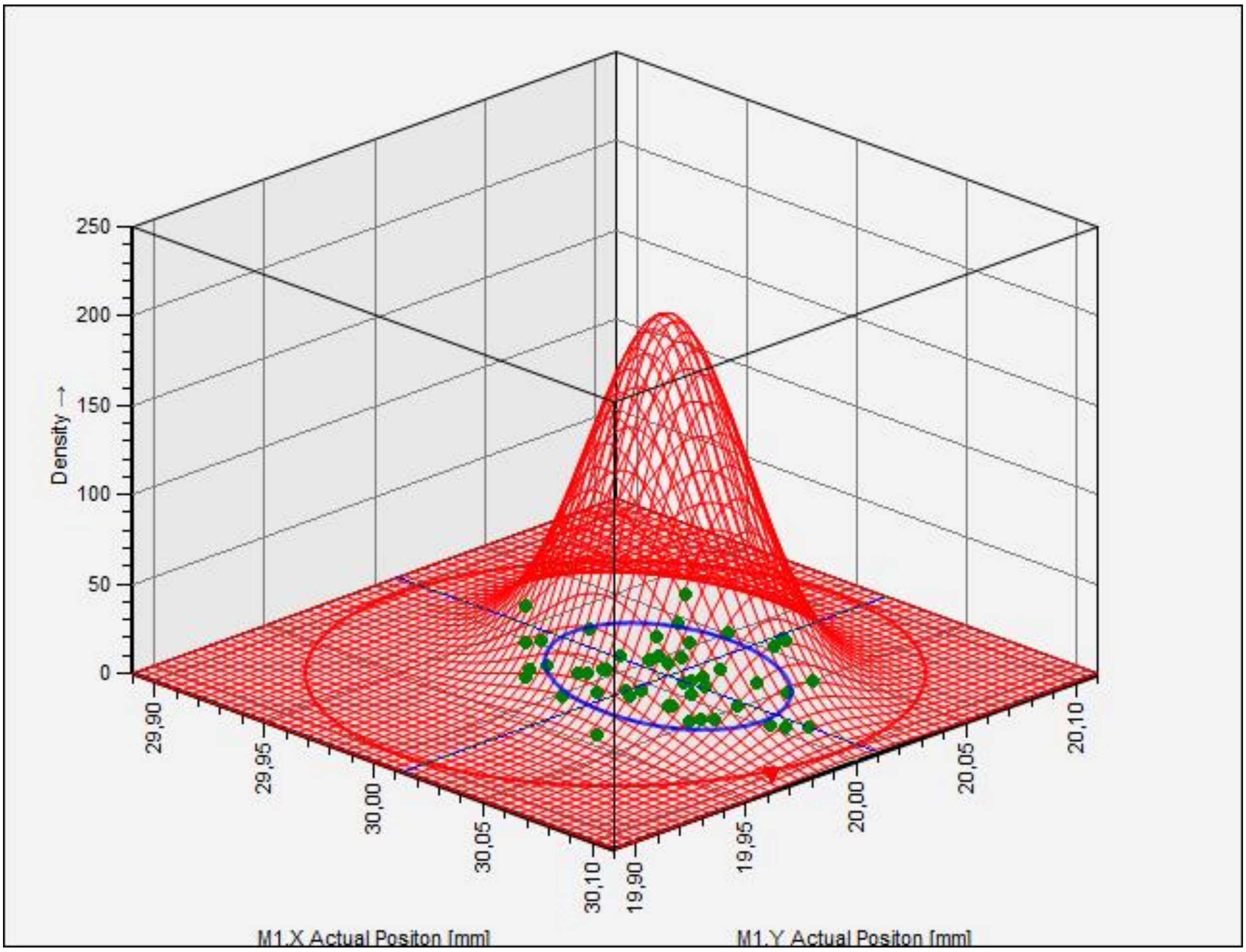


Figure 3-9: 50 measurement results of the actual positions of boreholes with the fitted model of the two-dimensional normal distribution (light blue "grid mountain")

Figure 3-9 shows the measurement results of 50 borehole positions together with the fitted model of the two-dimensional normal distribution ("wireframe mountain"). The red circle represents the tolerance of the position deviation and has the diameter t_{PS}.

In the case of the two-dimensional normal distribution, the scatter range of the characteristic values is a scatter ellipse. In Figure 3-9 the 68.27 % random scatter range can be seen as a blue ellipse. Interpretation: As expected, 683 of 1,000 measured values lie within the circumference of the scatter ellipse

3.4.1 The two-dimensional normal distribution

The probability density function of the two-dimensional normal distribution is generally:

$$g(x;y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\left\{-\frac{1}{2(1-\rho^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}}$$

The mean vector of the sample

$$\hat{\mu} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

represents the estimator for the expected value of the two-dimensional normal distribution. Similarly, from the covariance matrix of the sample we obtain the estimator for the covariance matrix of the model distribution.

$$\hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_x^2 & \hat{\sigma}_{xy} \\ \hat{\sigma}_{xy} & \hat{\sigma}_y^2 \end{pmatrix} = \begin{pmatrix} s_x^2 & s_{xy} \\ s_{xy} & s_y^2 \end{pmatrix}$$

For drawing the contours of equal probability density of the 2D normal distribution at a given probability $P = 1 - \alpha$, we first determine the statistical distance of equal probability $k_{1-\alpha}$ using the following relation:

$$k_{1-\alpha} = \sqrt{\chi^2_{2;1-\alpha}} = \sqrt{2ln\left(\frac{1}{\alpha}\right)}$$

We then determine the lengths of the two ellipse semi-axes a and b as follows:

$$a = k \cdot \hat{\sigma}_v$$

$$b = k \cdot \hat{\sigma}_w$$

with

$$\hat{\sigma}_v = \sqrt{\frac{1}{2}\left(\hat{\sigma}_x^2 + \hat{\sigma}_y^2\right) + \frac{1}{2}\sqrt{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)^2 + 4\hat{\sigma}_{xy}^2}}$$

$$\hat{\sigma}_w = \sqrt{\frac{1}{2}\left(\hat{\sigma}_x^2 + \hat{\sigma}_y^2\right) - \frac{1}{2}\sqrt{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)^2 + 4\hat{\sigma}_{xy}^2}}$$

For drawing, we switch from the x-y-coordinate system to the v-w-coordinate system

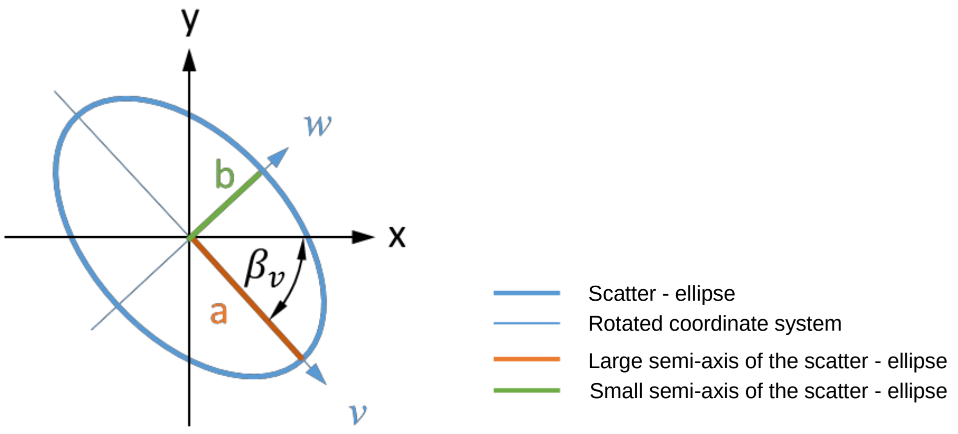


Figure 3-10: Sketch illustrating the scattering ellipse with the original reference frame (x-y-coordinate system) and the rotated reference frame (v-w-coordinate system) of the scattering ellipse

We determine the rotation angle between the x-axis of the old coordinate system and the v-axis of the new coordinate system as follows:

$$\beta_v = \frac{arctan\left(\frac{2\hat{\sigma}_{xy}}{(\hat{\sigma}_x^2 - \hat{\sigma}_y^2)}\right)}{2}$$

The inherent value of the covariance matrix Σ represent the variance in the direction of the two ellipse-major axes (v-w-coordinate system) and we determine them with:

$$\hat{\sigma}_v^2 = \frac{1}{2}(\hat{\sigma}_x^2 + \hat{\sigma}_y^2) + \frac{1}{2}\sqrt{(\hat{\sigma}_x^2 - \hat{\sigma}_y^2)^2 + 4\hat{\sigma}_{xy}^2}$$

$$\hat{\sigma}_w^2 = \frac{1}{2}(\hat{\sigma}_x^2 + \hat{\sigma}_y^2) - \frac{1}{2}\sqrt{(\hat{\sigma}_x^2 - \hat{\sigma}_y^2)^2 + 4\hat{\sigma}_{xy}^2}$$

3.4.1.1 Parameter estimation for 2D-NV for the example data set

Since within Section 3.4 the calculation results for the data of the case study "positional_deviation_example.dfq" are used repeatedly in several places, we summarize them here.

Expected values

$$\hat{\mu} = \begin{pmatrix} \hat{\mu}_x = \bar{x} \\ \hat{\mu}_y = \bar{y} \end{pmatrix} = \begin{pmatrix} 30{,}01376 \\ 20{,}01022 \end{pmatrix}$$

with

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \approx 30{,}01376mm\ ;\ \ \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \approx 20{,}01022mm$$

Covariance matrix

$$\hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_x^2 & \hat{\sigma}_{xy} \\ \hat{\sigma}_{xy} & \hat{\sigma}_y^2 \end{pmatrix} = \begin{pmatrix} 0{,}00107888 & -0{,}000145191 \\ -0{,}000145191 & 0{,}000589889 \end{pmatrix}$$

with the variance $\hat{\sigma}_x^2$, the variance $\hat{\sigma}_y^2$ and the covariance $\hat{\sigma}_{xy}$

$$\hat{\sigma}_x^2 = s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \approx 0{,}001\ 078\ 88\ mm^2$$

$$\hat{\sigma}_y^2 = s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \approx 0{,}000\ 589\ 889\ mm^2$$

$$\hat{\sigma}_{xy} = s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) \approx -0{,}000\ 145\ 191\ mm^2$$

The calculation of **the inherent values of the covariance matrix** leads to the variances of the 2D normal distribution in the direction of the axes of the rotated v-w -coordinate system

$$\hat{\sigma}_v^2 = \frac{1}{2}\left(\hat{\sigma}_x^2 + \hat{\sigma}_y^2\right) + \frac{1}{2}\sqrt{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)^2 + 4\hat{\sigma}_{xy}^2} \approx 0{,}001\ 118\ 741\ mm^2$$

$$\hat{\sigma}_w^2 = \frac{1}{2}\left(\hat{\sigma}_x^2 + \hat{\sigma}_y^2\right) - \frac{1}{2}\sqrt{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)^2 + 4\hat{\sigma}_{xy}^2} \approx 0{,}000\ 550\ 028\ mm^2$$

From the square root of these inherent values, we obtain the **standard deviations**:

$\hat{\sigma}_v \approx 0{,}033\,447\,582\ mm^2$

$\hat{\sigma}_w \approx 0{,}023\,452\,681\ mm^2$

The **correlation** between the variables x and y:

$$\hat{\varrho} = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y} \approx -0{,}181\,999$$

The **rotation angle β_v** between the x-axis of the original x-y-coordinate system and the v-axis of the new v-w-coordinate system is:

$$\beta_v = \frac{arctan\left(\frac{2\hat{\sigma}_{xy}}{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)}\right)}{2} \approx\ -0{,}267\,938\,574\ rad$$

By multiplying by $\frac{180}{\pi}$ we determine the angels in the unit degrees:

$$\beta_v \approx -15{,}35\ °$$

3.4.2 Determination of the parameter Po according to DIN ISO 22514-6

With the characteristic value P_o **we assess whether the dispersion of** the position measurement values **is basically small enough**. The main difference in determining the characteristic value $P_o$compared to $P_{ok}$is the fact that the actual mean position is ignored: mentally we shift the process distribution with its mean value exactly to the nominal position. Using the parameter $P_{O,}$we determine whether the positions now scattering around the nominal value can *basically* be generated "safely" within the tolerance circle with the diameter *tps*.

Source of the procedure: The procedure described here corresponds to **Type I** according to **ISO 22514-6: 2013-02**, which is referred to in the **software** as "**MPo2 max. probabilitysellipse**".

We will look at the procedure here in step sequences.

Step 1 - Shifting the Normal Distribution: We shift the two-dimensional normal distribution from the current mean position to the target value. The shift is illustrated by a small red arrow in Figure 3-11.

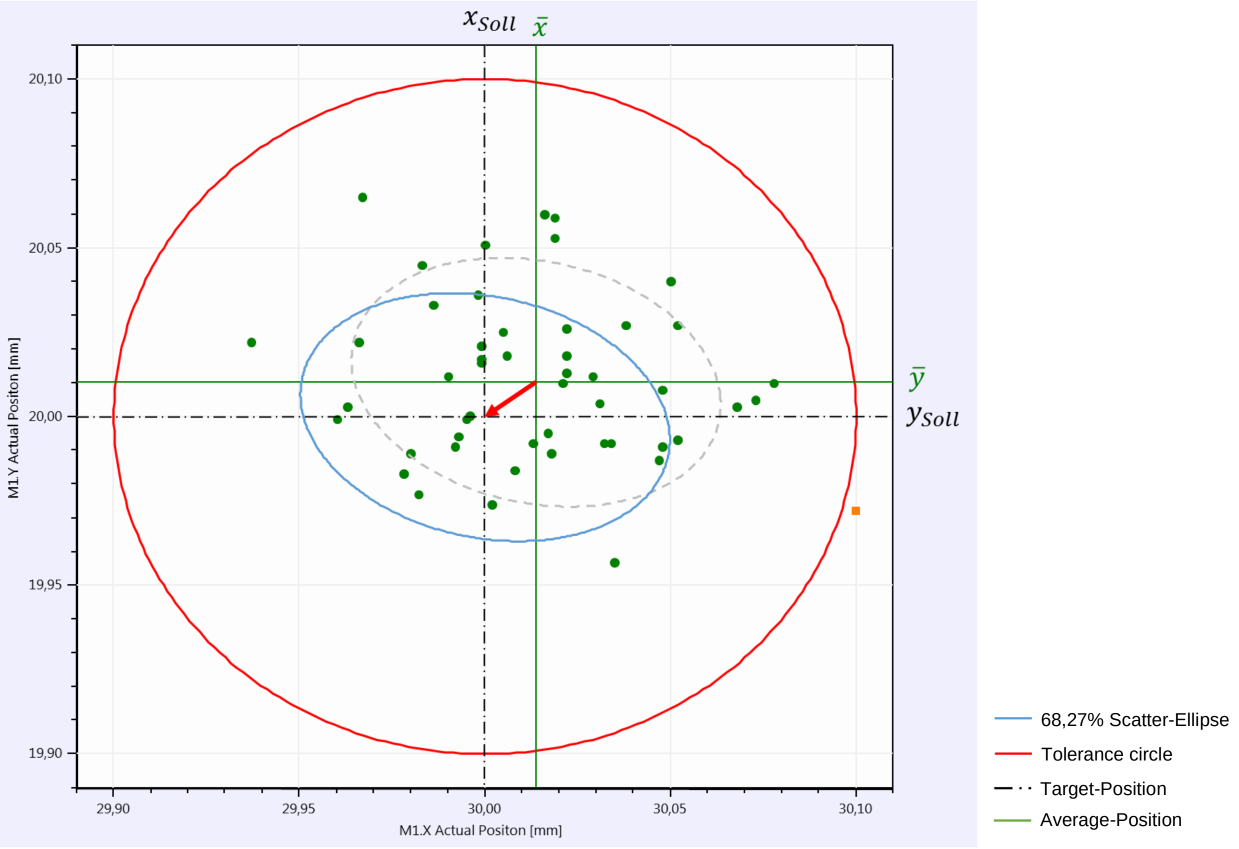


Figure 3-11: Representation of the Po ellipse for the position measurement values of the case study

Due to this shift, the process distribution is now in the "ideal position": Exactly centered on the setpoint.

Step 2 - Determine the probability α: In this step we are looking for the probability α below the 2D normal distribution, which is **outside the** P_o-Ellipse shown in Figure 3-12:

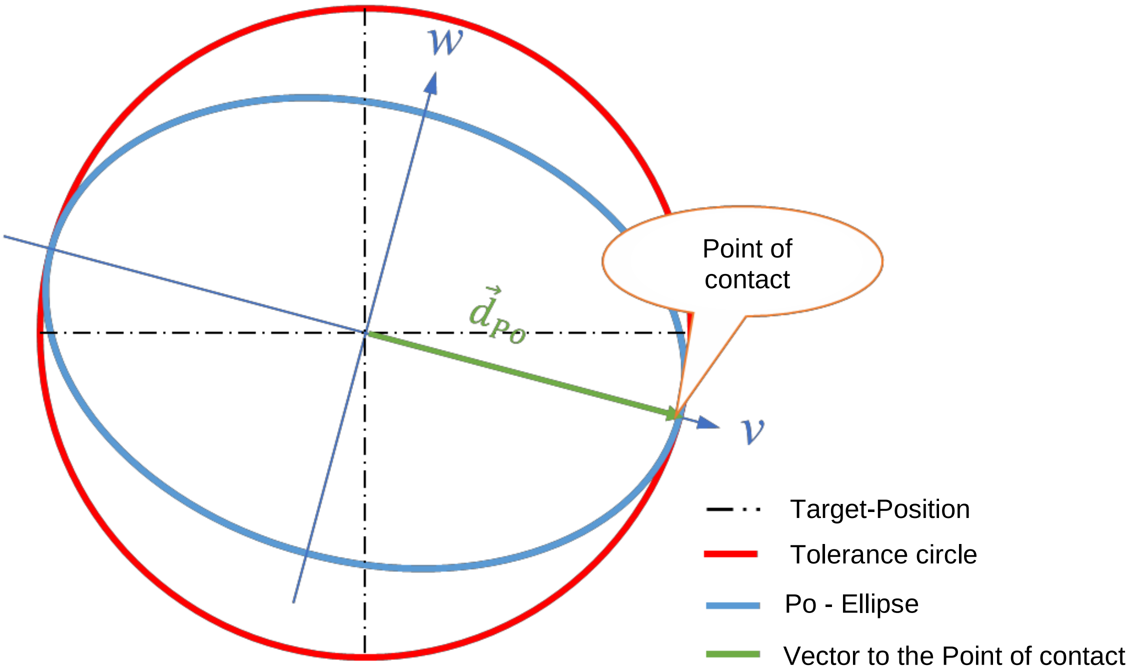


Figure 3-12: Illustration of the scattering ellipse just touching the tolerance circle and the vector $\vec{d}_{Po}$ pointing to the point of contact

First, we determine the minimum statistical distance[5] between the nominal position and the tolerance circle point of contact:

$$k_{Po} = \sqrt{\left(\frac{v_{Po}}{\sigma_v}\right)^2 + \left(\frac{w_{Po}}{\sigma_w}\right)^2}$$

In Figure 3-12, the vector $\vec{d}_{Po}$ is shown in green. This vector has the coordinates $v_{Po} = \frac{t_{PS}}{2}$ and $w_{Po} = 0$. For these coordinates, the smallest statistical distance to the tolerance circle is given.

[5] What we refer to here as the "statistical distance" is often referred to in the literature as the "Mahalanobis distance".

Using the data from the example, this gives us the following result for the minimum statistical distance:

$$k_{Po} = \sqrt{\left(\frac{v_{Po}}{\sigma_v}\right)^2 + \left(\frac{w_{Po}}{\sigma_w}\right)^2} \approx \sqrt{\left(\frac{0{,}1\,mm}{0{,}033\,447\,583\,mm}\right)^2 + 0} \approx \frac{0{,}1\,mm}{0{,}033\,447\,583\,mm} \approx 2{,}989\,753$$

with

$$v_{Po} = \frac{t_{PS}}{2} = \frac{0{,}2\,mm}{2} = 0{,}1\,mm$$

$$\hat{\sigma}_v = \sqrt{\frac{1}{2}\left(\hat{\sigma}_x^2 + \hat{\sigma}_y^2\right) + \frac{1}{2}\sqrt{\left(\hat{\sigma}_x^2 - \hat{\sigma}_y^2\right)^2 + 4\hat{\sigma}_{xy}^2}} \approx 0{,}033\,447\,6\,mm$$

$$\hat{\sigma}_x^2 = s_x^2 = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right] mm^2 \approx 0{,}001\,078\,88\,mm^2$$

$$\hat{\sigma}_y^2 = s_y^2 = \left[\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2\right] mm^2 \approx 0{,}000\,589\,889\,mm^2$$

$$\hat{\sigma}_{xy} = s_{xy} = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right] mm^2 \approx -0{,}000\,145\,191\,mm^2$$

Finally, we compute the probability volume α of the 2D normal distribution, which is

is **outside the $\boldsymbol{P_O}$-Ellipse**.

$$\alpha = e^{-\left(\frac{1}{2}k_{Po}^2\right)} \approx 0{,}011\,455\,21$$

Note: We must note that the probability α calculated here describes the fraction outside the P_O-Ellipse and **not** the expected fraction of exceedance outside the tolerance circle with exact process centering. For this reason, we **cannot** use the probability α to estimate the expected fraction of nonconforming units under exact process centering

Step 3 - Determine the probability Ppo:

Now a "reinterpretation" of the probability volume α takes place:

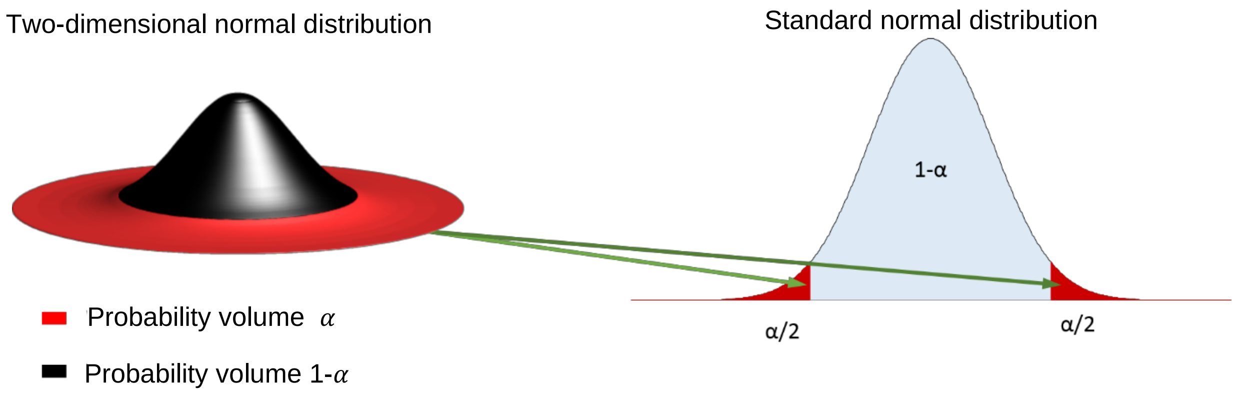


Figure 3-13: Illustration of how the probability volume α of the two-dimensional normal distribution is reinterpreted as the two-sided excess part of a one-dimensional standard normal distribution

We think of the probability volume α below the two-dimensional normal distribution as a two-sided split excess portion of a one-dimensional standard normal distribution (see Figure 3-13).

In Figure 3-14, the probability $\alpha/2$ is shown in red and the complementary probability $P_{Po} = 1 - \frac{\alpha}{2}$ is shown in pale blue

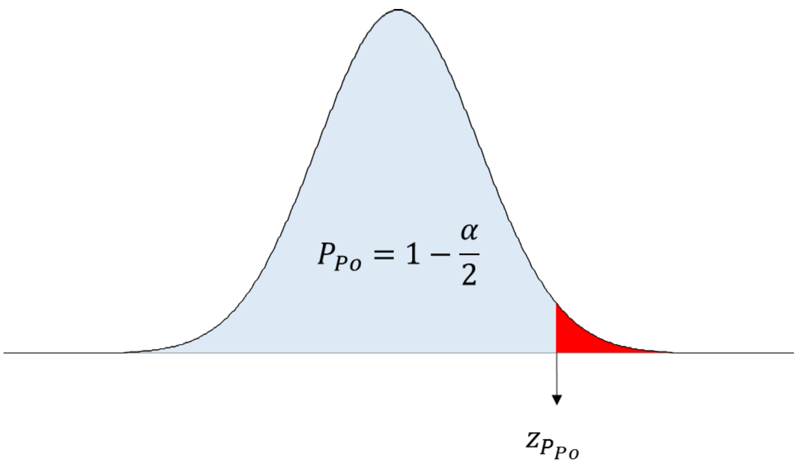


Figure 3-14: Standard normal distribution with probability P_{Po} and associated quantile $z_{P_{Po}}$

The value of the probability P_{po} we are looking for is

$P_{Po} = 1 - \frac{\alpha}{2} \approx 0{,}994\ 272\ 4$

Step 4 - Determine the quantile of the one-dimensional standard normal distribution:
We calculate the quantile to *probability* P_{Po} using the inverse distribution function (quantile function) of the one-dimensional standard normal distribution

$$z_{P_{PO}} = G^{-1}(P_{Po} = 0{,}994\,272\,4) \approx 2{,}528\,497$$

with

G^{-1} = inverse distribution function of the one-dimensional standard normal distribution

Step 5 - Calculation of the capability parameter P_o:

To determine this parameter, we divide the quantile calculated in step 4 by three.

$$P_O = \frac{z_{P_{PO}}}{3} \approx \frac{2{,}528\,497}{3} \approx 0{,}84$$

The requirement is considered satisfied if the calculated P_o-value is greater than or equal to the specified minimum $P_{o_{min}}$

$$P_o \geq P_{o_{min}}$$

3.4.3 Determination of the parameter P_{ok} according to DIN ISO 22514-6

With the parameter P_{ok} **we judge whether** the **scatter of the** (model) distribution of the position measurements, **taking into account the actual mean value position, is small enough to be** able to produce the positions "safely" within the tolerance circle.

Source of the procedure: The procedure described here corresponds to **Type I** according to **ISO 22514-6: 2013-02.** This procedure is referred to in the **software** as "**MPo2 max. probability ellipse**".

Compared to Section 3.4.2, only the **first step is** identical, in which the determination of the **parameter estimators** for the two-dimensional normal distribution takes place.

Step 2 - Determine the probability α for the scattering ellipse touching the tolerance circle:
From Figure 3-15 we can see that the normal distribution (and thus the ellipse) is not in the nominal position. The actual mean position is used and thus taken into account.

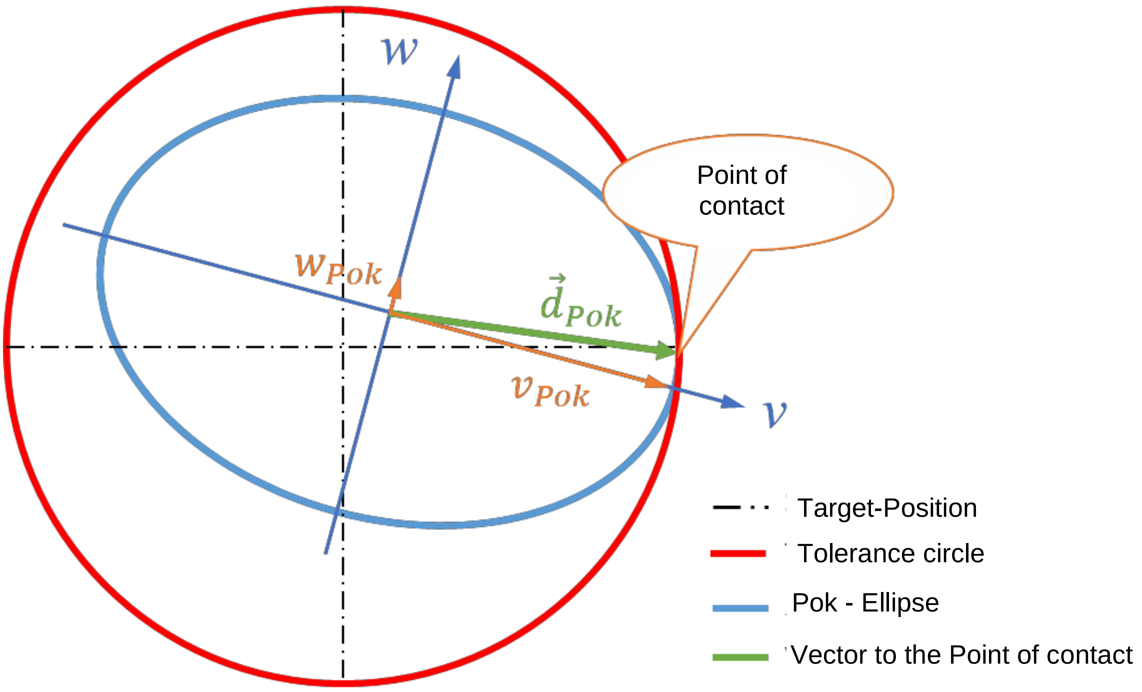


Figure 3-15: Illustration of the P_{ok}--Ellipse showing the distances v_{Pok} and w_{Pok}

The probability volume $\alpha*$ of the two-dimensional normal distribution that is outside the scattering ellipse touching the tolerance circle (see Figure 3-15) is sought.

To do this, first determine the minimum statistical distance[6] k_{Pok} between the ellipse center and the tolerance circle:

$$k_{Pok} = \sqrt{\left(\frac{v_{Pok}}{\hat{\sigma}_v}\right)^2 + \left(\frac{w_{Pok}}{\hat{\sigma}_w}\right)^2}$$

In the Figure 3-15, in green color, the vector $\vec{d}_{Pok}$ is shown. With the coordinates v_{Pok} and w_{Pok} of this vector, the minimum statistical distance to the tolerance circle is obtained. The procedure for determining the minimum statistical distance is more complex, so we have moved the presentation to the appendix and omit the details of the determination here. For the data of the case study we obtain the minimum statistical distance

$$k_{Pok} \approx 2{,}625\ 029$$

[6] What we refer to here as "statistical distance" is often called "Mahalanobis distance.".

Using the minimum statistical distance k_{Pok}, we compute the probability volume $\alpha*$ that is **outside the P_{ok}-Ellipse**:

$\alpha^* = e^{-\left(\frac{1}{2}k_{Po}^2\right)} \approx e^{-\left(\frac{1}{2}2{,}625\,029^2\right)} \approx 0{,}031\,892$

Note: Again, it should be noted that the calculated probability describes the expected fraction outside the P_{ok}-Ellipse. Thus, this probability is **not** the expected exceedance fraction outside the tolerance circle. For this reason, we **cannot** use the probability α^* to estimate the expected fraction of nonconforming units

Step 3 - Determine the probability $P_{P_{ok}}$

Again, the "reinterpretation" of the probability α^* as a two-sided split excess fraction of a one-dimensional standard normal distribution is done, quite analogous to the description for steps three and four in Section 3.4.2. We determine the probability we are looking for as follows

$P_{Pok} = 1 - \frac{\alpha^*}{2} \approx 0{,}984\,054$

Step 4 - Determine the quantile $z_{P_{Pok}}$

Using the inverse distribution function of the one-dimensional standard normal distribution (quantile function), we calculate the quantile $z_{P_{Pok}}$:

$z_{P_{Pok}} = G^{-1}(P_{Pok}) \approx 2{,}145\,757$

Step 5 - Calculate the minimum capability index P_{ok}

We obtain the minimum ability index P_{ok} by dividing the quantile $z_{P_{Pok}}$ by the value 3

$P_{ok} = \frac{z_{P_{Pok}}}{3} \approx 0.715\,252 \approx 0{,}72$

The requirement is considered to be met if the minimum capability index is greater than or at least equal to the specified minimum value:

$P_{ok} \geq P_{ok_{min}}$

The following result was generated with the program qs-STAT in the module "Sample Analysis" based on the evaluation strategy "Q-DAS Machine Capability (06/2013)":

Part no.	Ex PD	Part descr.	Positional Deviation Example
Char.No.	PD	Char.Descr.	Positional Deviation

Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07632
T*	0.200	R	0.200	$X_{0.135\%}$	-0.00081 [rt]
Characteristics C :	unimportant	n_{eff}	50	$X_{99.865\%}$	0.23057 [rt]
		n_{tot}	50	X_{up3}-X_{lo3}	0.23138 [rt]
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.32245 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.67755%
		$n_{<LSL}$	---	$p_{<LSL}$	---

Model distribution			Weibull distribution
Distr.regress.coeff.	r_{tot}	:	0.99559428
Distr.regress.coeff.	$r_{25\%}$	:	0.97230551
Calculation method			MPo2 max. probability ellipse
Potential Capability index	P_o	$0.68 \le \mathbf{0.84} \le 1.01$	0 … 1.67
Critical capability index	P_{ok}	$0.55 \le \mathbf{0.72} \le 0.88$	0 … 1.67
The requirements were not met (P_o,P_{ok})			
Demand Potential Capability index	$P_{o\ target}$	1.67	
Demand Critical capability index	$P_{ok\ target}$	1.67	
Q-DAS Machine Capability (06/2013)			

Figure 3-16: Window "Form - Display 3" with the calculation result according to the calculation method "MPo2 max. probability ellipse" for the example data set "positional_deviation_example.dfq" (call with function key F10)

3.5 Calculation type "MPo3 max. probability ellipse / line"

This method is roughly related to the "MPo2 max. probability ellipse" calculation method, but avoids its numerical disadvantages. That is, here we bypass the determination of the probability volume below the 2D normal distribution and also the subsequent determination of the inverse distribution function (quantile) of the one-dimensional standard normal distribution.

3.5.1 Determining the parameter Po

In the **first step** we estimate all parameters according to section 3.4.1. In this procedure we also need the minimum statistical distance k_{Po} between the ellipse center and the tolerance circle. Here, the **ellipse center** is **shifted to the nominal position**.

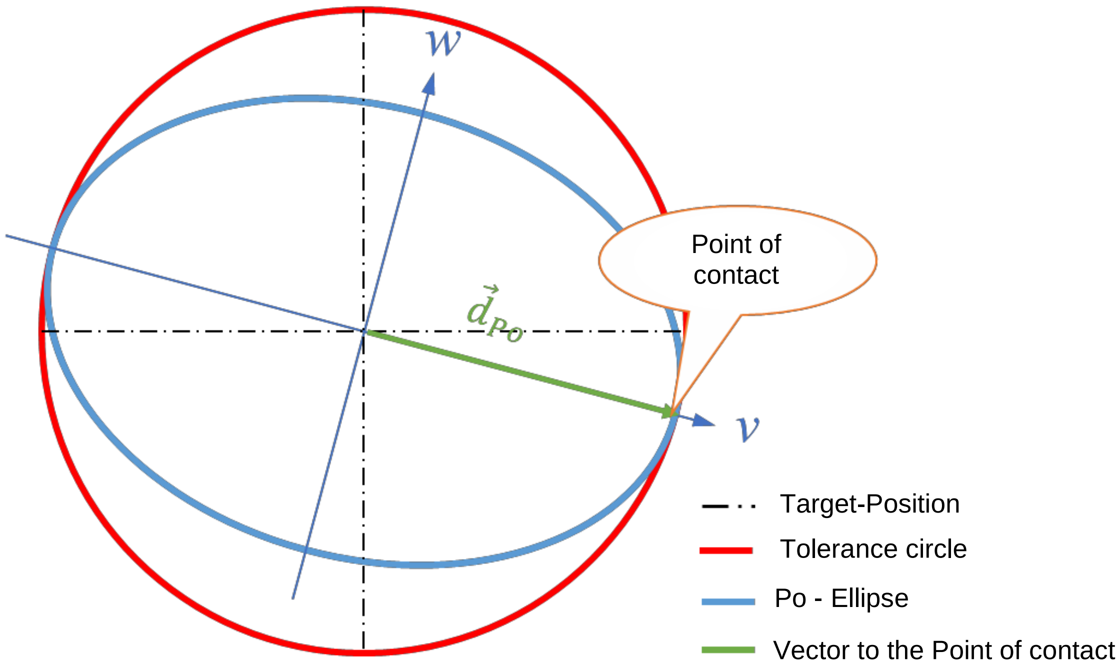


Figure 3-17: Illustration of the scattering ellipse touching the tolerance circle with the vector $\vec{d}_{Po}$ pointing to the point of contact, whose vector coordinates are $v_{Po} = \frac{t_{PS}}{2}$ and $w_{Po} = 0$

Figure 3-17 shows the vector $\vec{d}_{Po}$ pointing to the point of contact. This vector has the coordinates $v_{Po} = \frac{t_{PS}}{2} = 0{,}1mm$ and $w_{Po} = 0$. With these coordinates we determine the minimum statistical distance kPo

$$k_{Po} = \sqrt{\left(\frac{v_{Po}}{\hat{\sigma}_v}\right)^2 + \left(\frac{w_{Po}}{\hat{\sigma}_w}\right)^2} \approx \sqrt{\left(\frac{0{,}1\ mm}{0{,}033\ 447\ 583\ mm}\right)^2 + 0} \approx \frac{0{,}1\ mm}{0{,}033\ 447\ 583\ mm} \approx 2{,}989\ 753$$

Now we divide the minimum statistical distance k_{Po} by three and get the performance index $\boldsymbol{P_o}$:

$$P_O = \frac{k_{Po}}{3} \approx \frac{2{,}989\ 753}{3} \approx 0{,}996\ 584\ \approx 1{,}00$$

3.5.2 Determination of the parameter Pok

To determine the parameter P_{ok} we need the minimum statistical distance between the ellipse center and the tolerance circle contact point. The **ellipse center** is **not shifted** and corresponds to the mean value of the observation data. Figure 3-18 shows the scatter ellipse just touching the tolerance circle.

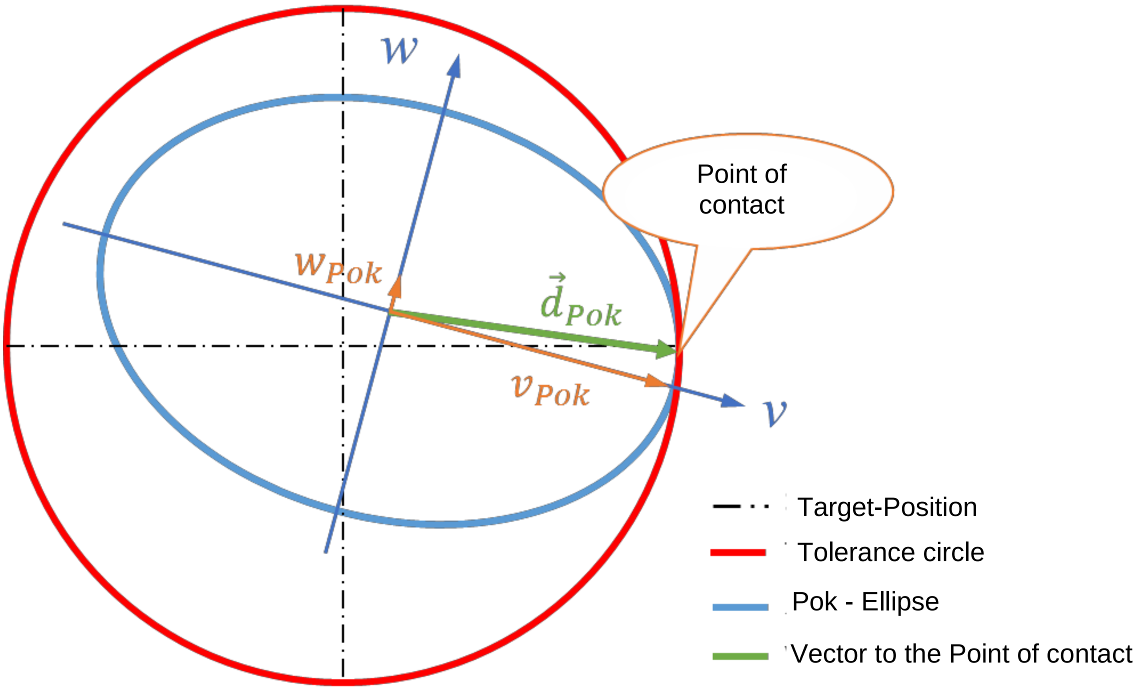


Figure 3-18: Illustration of the vector d_{Pok} pointing to the touch point with vector coordinates v_{Pok} and w_{Pok}

The vector d_{Pok} points to the touch point and has vector coordinates w_{Pok} and v_{Pok}.. Using these vector coordinates, we calculate the minimum statistical distance k_{Pok}:

$$k_{Pok} = \sqrt{\left(\frac{v_{Pok}}{\sigma_v}\right)^2 + \left(\frac{w_{Pok}}{\sigma_w}\right)^2} \approx 2{,}625\ 029$$

The details of how to determine the minimum statistical distance is moved to the appendix due to greater complexity.

Dividing the minimum statistical distance k_{Pok} by the value 3, we obtain the **minimum performance index P_{ok}**:

$$P_{ok} = \frac{k_{Pok}}{3} \approx \frac{2{,}625\ 029}{3} \approx 0{,}875\ 010 \ \approx 0{,}88$$

The following evaluation result was created with the program "qs-STAT" in the module "Sample Analysis" on the basis of the evaluation strategy "Q-DAS Machine Capability (01/2020)".

Part no.	Ex PD	Part descr.	Positional Deviation Example
Char.No.	PD	Char.Descr.	Positional Deviation

Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07126
T*	0.200	R	0.200	$X_{0.135\%}$	0.00258
Characteristics C :	unimportant	n_{eff}	50	$X_{99.865\%}$	0.23648
		n_{tot}	50	X_{up3}-X_{lo3}	0.23391
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.19524 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.80476%
		$n_{<LSL}$	---	$p_{<LSL}$	---

Model distribution			Weibull distribution
Distr.regress.coeff.	r_{tot}	:	0.99618009
Distr.regress.coeff.	$r_{25\%}$	:	0.96951722
Calculation method			][MPo3 min. statistical distance
Potential Capability index	P_o	0.82 ≤ **1.00** ≤ 1.28	0 … 1.67
Critical capability index	P_{ok}	0.72 ≤ **0.88** ≤ 1.13	0 … 1.67
The requirements were not met (P_o,P_{ok},LV)			
Demand Potential Capability index	$P_{o\ target}$		1.67
Demand Critical capability index	$P_{ok\ target}$		1.67
⊕ Q-DAS Machine Capability (01/2020)			

Figure 3-19: Window "Form - Display 3" with the result of the capability calculation according to the calculation type "Q-DAS Machine Capability (01/2020)" for the data set "Postional_Deviation_Example.dfq" (call with function key F10)

3.6 Method MPo A1 [AFNOR E60-181]

This method is **only** available in the **process capabilityanalysis module** and is described in the French standard AFNOR E 60-181: 01-2001, section 4.7.8.

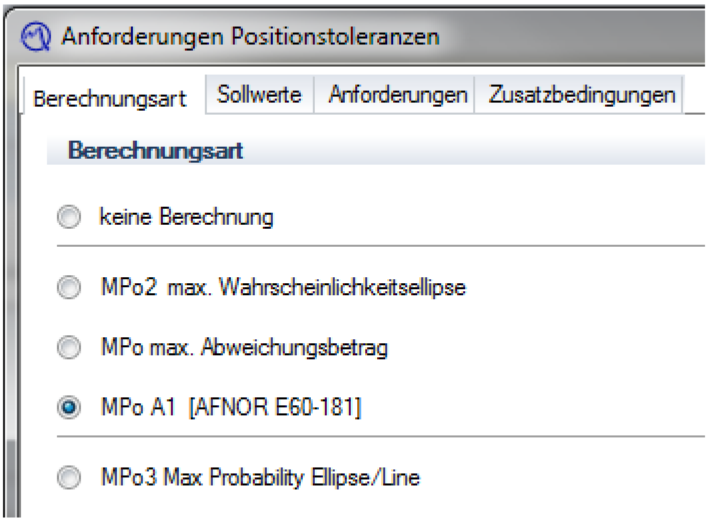


Figure 3-20: Calculation type MPo A1 [AFNOR E60-181] in the Process Analysis module

The capability calculation is based on the deviation amounts from the mean. That is, in the **first step** we determine the **Euclidean distances to the mean**:

$$r_i = \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}; i = 1,2, \dots, n$$

In the **second step,** we calculate the **mean value for the deviation amounts and** the **Standard deviation**:

$$\bar{\mathrm{r}} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

$$s_p = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2}$$

In the **third step,** we calculate the **performance index Cap**:

$$Cap = \frac{TG}{D_p}$$

Mit

$$TG = \frac{t_{PS}}{2}$$

$$D_p = 5.55 \cdot s_p$$

In the **fourth** and final **step,** we calculate the **minimum performance index Cpk**:

$$C_{pk} = \frac{(TG - \bar{r})}{D_p}$$

For the data from the example dataset "*positional_deviation_example.dfq"* we get:

$$Cap = \frac{TG}{D_p} = \frac{0{,}1\ mm}{0{,}105\ 464\ 887\ mm} \approx 0{,}948$$

$$C_{pk} = \frac{(TG - \bar{r})}{D_p} = \frac{(0{,}1 - 0{,}035\ 798\ 242)mm}{0{,}105\ 464\ 887\ mm} \approx 0{,}609$$

mit

$$\bar{\mathrm{r}} = 0{,}035\ 798\ 242\ mm$$

$$s_p = 0{,}019\ 002\ 682\ mm$$

$$D_p = 5.55 \cdot s_p = 0{,}105\ 464\ 887\ mm$$

$$TG = \frac{t_{PS}}{2} = \frac{0{,}2\ mm}{2} = 0{,}1\ mm$$

The following result was created with the program qs-STAT in the module "Process Analysis" on the basis of the evaluation strategy "AFNOR E 60-181".

Part no.	Ex PD	Part descr.		Positional Deviation Example	
Char.No.	PD	Char.Descr.		Positional Deviation	
Drawing Values		Collected Values		Statistics	
T_m	0.100	$\tilde{x}$	0.0690	$\bar{x}$	0.07680
LSL*	0.000	x_{min}	0.008	s	0.0431
USL	0.200	x_{max}	0.208	$X_{50\%}$	0.07680
T*	0.200	R	0.200	$X_{0.135\%}$	-0.05242
Characteristics C :	unimportant	n_{eff}	50	$X_{99.865\%}$	0.20602
		n_{tot}	50	6s	0.25844
		$n_{<T>}$	49 / 98.00000%	$p_{<T>}$	99.78832 %
		$n_{>USL}$	1/2.00000%	$p_{>USL}$	0.21168%
		$n_{<LSL}$	---	$p_{<LSL}$	---
Model distribution				Normal Distribution	
Distr.regress.coeff.		r_{tot}	:	0.98721731	
Distr.regress.coeff.		$r_{25\%}$	:	0.97277545	
Calculation method			MPo A1 [AFNOR E60-181]		
Potential Capability index		CAP	0.95	0	1.66
Critical capability index		CPK	0.61	0	1.66
The requirements were not met (CAP ,CPK ,P_{oi})					
Demand Potential Capability index		CAP_{target}		1.66	
Demand Critical capability index		CPK_{target}		1.66	
⊕ AFNOR E 60-181					

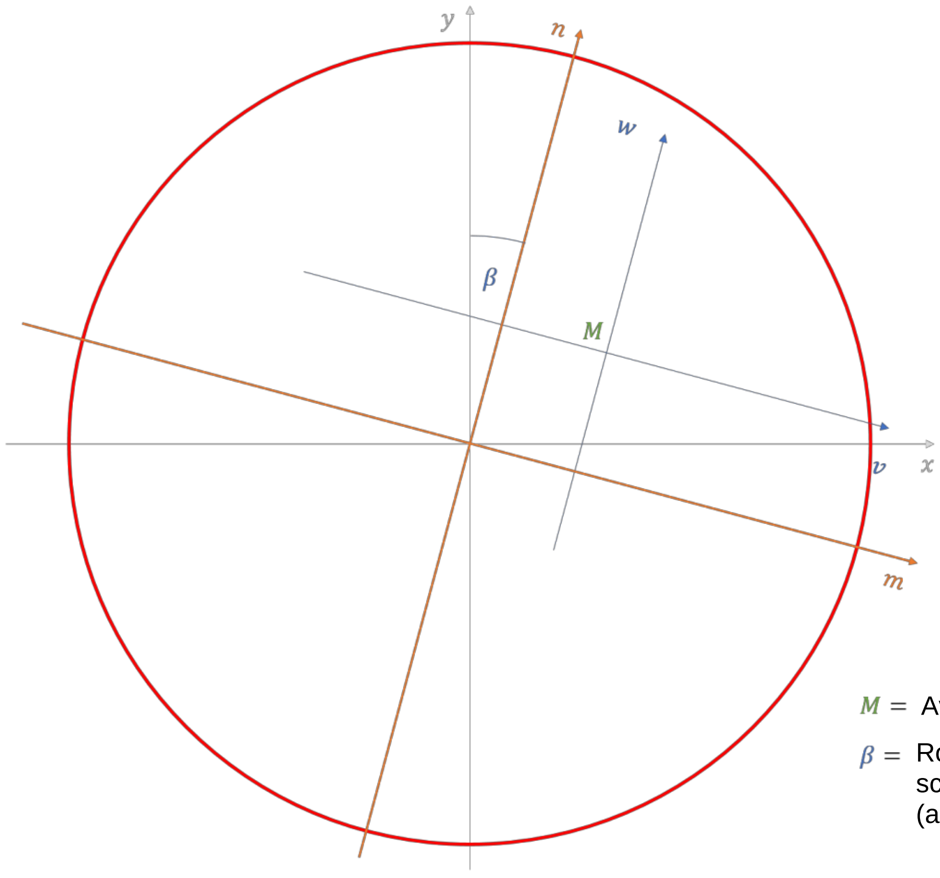
Figure 3-21: Window "Form - Representation 3" with the evaluation result according to the method MPo A1 [AFNOR E60-181] for the example data set "positional_deviation_example.dfq" (call with function key F10)

4 Appendix

4.1 Determine the statistical distance for the P_{ok}-Ellipse

We consider here the details about the second step from the Pok calculation, where the minimum statistical distance k between the mean of the position measurement results M and the tolerance circle has to be determined.

We need to find the exact point on the circumference of the circle that has the *smallest statistical* distance to the point M. What makes life difficult for us in this search is the fact that the position of the v-w-coordinate system is shifted and twisted with respect to the x-y-coordinate system.



$M =$ Average

$\beta =$ Rotation angle of the scattering ellipse (and the new C' - system)

Figure 4-1: The original x-y-coordinate system is rotated so that it is oriented at the same angle to the v-w-coordinate system.

4.1.1 Rotation of the coordinate system

To make it easier for our calculations, we first rotate the original x-y- coordinate system so that it is exactly aligned with the v-w-coordinate system of the scattering ellipse. We refer to the resulting new coordinate system here as the n-m-coordinate system. This rotation results in different coordinate values for the mean M in the new n-m-coordinate system than in the old x-y-coordinate system. The next three sections show the determination of these new n- and m-coordinate values for the point M.

4.1.2 Determine the vector from the target position to the mean position *M*

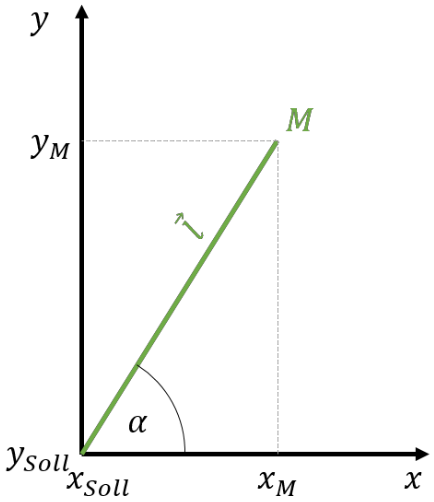


Figure 4-2: Vector l in the original x-y coordinate system

We already know the coordinates of the point M from our mean (Section 3.4.1) in the original x-y coordinate system.

$$x_M = 30.013\ 76\ mm$$

$$y_M = 20{,}010\ 22\ mm$$

and the nominal position (section 1.1):

$$x_{\text{tar}} = 30{,}000\,mm$$

$$y_{\text{tar}} = 20{,}000\,mm$$

Mit diesen Informationen berechnen wir den Betrag $|\vec{l}|$ und den Winkel α des Vektors $\vec{l}$

With this information, we calculate the magnitude and angle α of the vector

$$|\vec{l}| = \sqrt{(x_M - x_{tar})^2 + (y - y_{tar})^2}$$

$$|\vec{l}| = \sqrt{(30{,}013\ 76\ mm - 30{,}000\ 00\ mm)^2 + (20{,}010\ 22mm - 20{,}000\ 00\ mm)^2}$$

$$|\vec{l}| = \sqrt{0{,}013\ 76^2 mm^2 + 0{,}010\ 22^2 mm^2} = 0{,}017\ 140\ 187\ mm$$

$$\alpha = arctan\frac{(x_M - x_{tar})}{(y_M - y_{tar})}$$

$$\alpha = arctan\frac{(30{,}013\ 76\ mm - 30{,}000\ 00\ mm)}{(20{,}010\ 22\ mm - 20{,}000\ 00\ mm)} = 0{,}638\ 833\ 7\ rad$$

4.1.3 Determine the angle $\boldsymbol{\delta}$ of the vector $\vec{l}$ in the $\boldsymbol{n}$-$\boldsymbol{m}$-coordinate system

Figure 4-3 shows the angle δ of the vector $\vec{l}$ .This angle defines the location of the vector in the new n-m coordinate system. Now we determine the value of this angle:

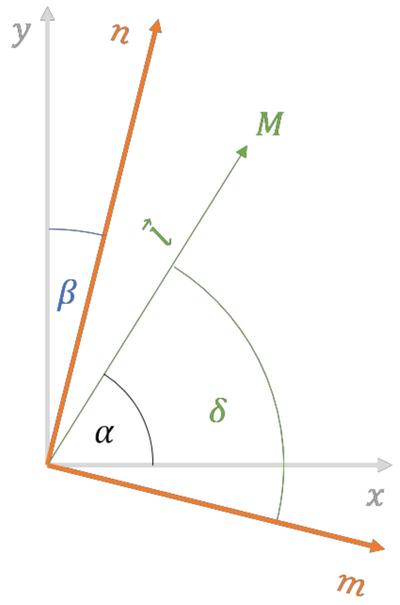


Figure 4-3: Plot of the angle δ of the vector $\vec{l}$ in the n-m coordinate system

It is known from previous calculations:

$\alpha = 0{,}638\ 833\ 7\ rad$

$\beta = -0{,}267\ 938\ 574\ rad$

With these values, we calculate the angle δ considering the mathematical direction of rotation (counterclockwise).

Angle of the vector l in the m-n coordinate system: $\delta = \alpha - \beta$

$\delta = 0{,}638\ 833\ 7\ rad - (-0{,}267\ 938\ 574\ rad) = 0{,}906\ 772\ 27\ rad$

4.1.4 Coordinates of the vector $\vec{l}$ in the $\boldsymbol{n}$-$\boldsymbol{m}$-coordinate system

Now we determine the coordinates n_M and m_M for our mean (point M) in the new n-m coordinate system

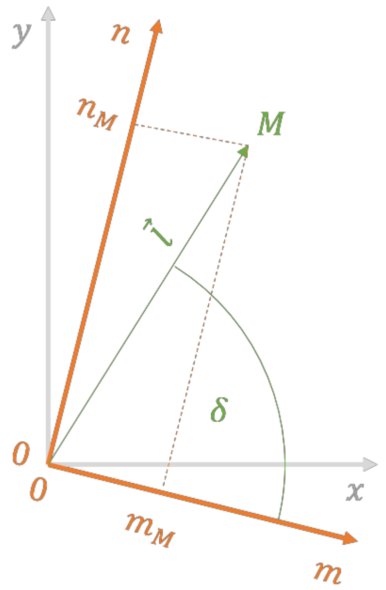


Figure 4-4: Illustration of the coordinates m_M and n_M

Note: To simplify the calculations, the target position in the m-n-coordinate system was simply set to zero. The target position is shown as a point (0 | 0) in Figure 4-4.

Known from previous calculations:

Amount of the vector $\vec{l}$ $\quad |\vec{l}| = 0{,}017\ 140\ 187\ mm$

Angel of the vector $\vec{l}$ $\quad \delta = 0{,}906\ 772\ 27\ rad$

Coordinate of the vector $\vec{l}$ in the direction of the m-axis

$$m_M = |\vec{l}| \cdot \cos\delta$$

$$m_M = 0{,}017\ 140\ 187\ mm \cdot \cos(0{,}906\ 772\ 27\ rad)$$

$$m_M = 0{,}010\ 563\ 34\ mm$$

Coordinate of the vector l in the direction of the n-axis:

$$n_M = |\vec{l}| \cdot \sin \delta$$

$$n_M = 0{,}017\,140\,187\,mm \cdot sin(0{,}906\,772\,27\,rad)$$

$$n_M = 0{,}013\,498\,22\,mm$$

4.1.5 Determining the statistical distance k from the mean value M to the tolerance circle

Now consider determining the statistical distance k between the mean position M and any point P on the perimeter of the tolerance circle.

To determine any point P on the circumference of the tolerance circle, we need the circle equation:

$$r^2 = m_P^2 + n_P^2$$

With $r = \frac{t_{PS}}{2} = \frac{0,2\,mm}{2} = 0,1\,mm$

To obtain the coordinates for any point P on the perimeter of the tolerance circle, we first choose a value for the variable m_P (in the interval $-r \leq m_P \leq r$).

We then determine the associated value of the variable nP using the circular formula:

$$n_p = \pm\sqrt{r^2 - m_P^2}$$

Looking at Figure 4-5, we see that the smallest statistical distance for this case study is to be found in the first quadrant of the m-n-coordinate system (So, thought of as the hand position of a clock, it is in the range between 12:00 and 03:00):

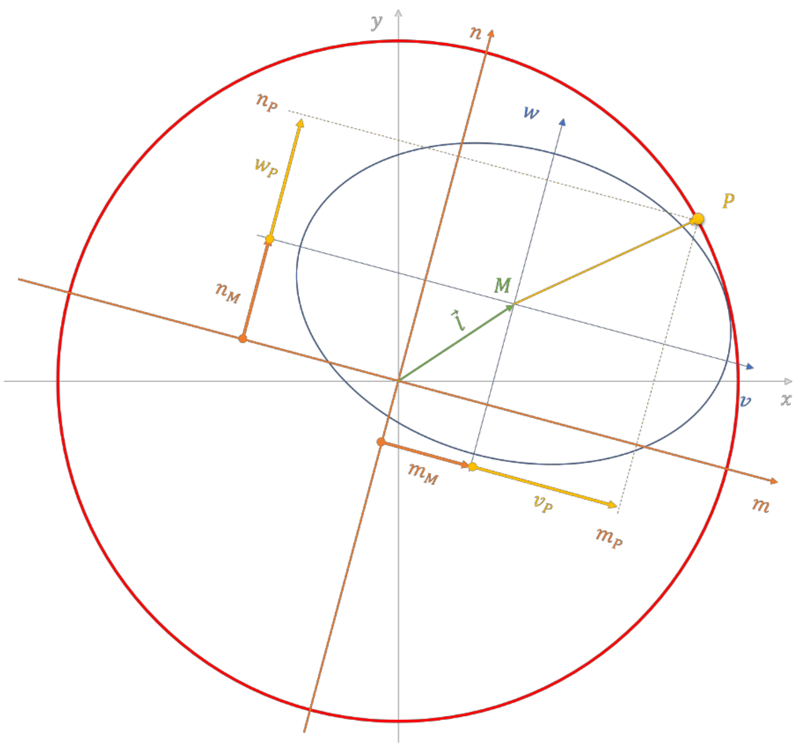


Figure 4-5: Representation of the vector from the point M to the point P with all associated vector components

Further, we can see from Figure 4-5 that the statistical distance for the distance between the mean M and the point P must be calculated using the vector components vp and wp in the v-w coordinate system of the scattering ellipse.

Therefore, we now switch to the v-w coordinate system:

$$v_P = m_P - m_M$$

$$w_P = n_P - n_M$$

Using these coodinate values, we calculate the statistical distance k from the point M to the point P.

Statistical distance k:

$$k = \sqrt{\left(\frac{v_P}{\hat{\sigma}_v}\right)^2 + \left(\frac{w_P}{\hat{\sigma}_w}\right)^2}$$

But for which point P on the tolerance circle does the smallest statistical distance to the point M result? Since we do not want to determine the value here using differential calculus, we choose the direct numerical search. To do this, for example, we generate 100 values for the variable m in the interval from $m = mM$ to $m = r$. Then, for each value of the variable m, we determine the function value n:

$$n = +\sqrt{r^2 - m^2}.$$

Then we switch from the n-m-coodinate system to the v-w-coodinate system of the scattering ellipse. To do this, we determine the coordinate values vP and wP as follows:

$v_P = m - m_M$ und $w_P = n - n_M$

Finally, using the v_P – und w_P -coordinate values, we calculate the statistical distance k:

$$k = \sqrt{\left(\frac{v_P}{\hat{\sigma}_v}\right)^2 + \left(\frac{w_P}{\hat{\sigma}_w}\right)^2}$$

The calculation steps just described are summarized in the following table.
For the results shown therein, the following values, already known from the previous calculations, were used:
$m_M = 0{,}010\ 563\ 34\ mm$, $n_M = 0{,}013\ 498\ 22\ mm$, $\hat{\sigma}_v = 0{,}033\ 447\ 582\ mm$ und $\hat{\sigma}_w = 0{,}023\ 452\ 681\ mm$.

m	$n = \sqrt{0{,}1^2 - m^2}$	$v_p = m - m_M$	$w_p = n - n_M$	$k = \sqrt{\left(\frac{v_p}{\hat{\sigma}_v}\right)^2 + \left(\frac{w_p}{\hat{\sigma}_w}\right)^2}$
0,010 563 34	0,099 440 51	0,000 000 00	0,008 594 23	3,664 498
0,011 466 74	0,099 340 39	0,000 903 40	0,008 584 21	3,660 328
0,012 370 14	0,099 231 95	0,001 806 80	0,008 573 37	3,656 004
⋮	⋮	⋮	⋮	⋮
0,100 000 00	0,000 000 00	0,089 436 66	-0,001 349 82	2,735 176

If we graphically plot the statistical distance k over the values of the variable m, we obtain the following plot:

Statistical distance k as a function of the variables m

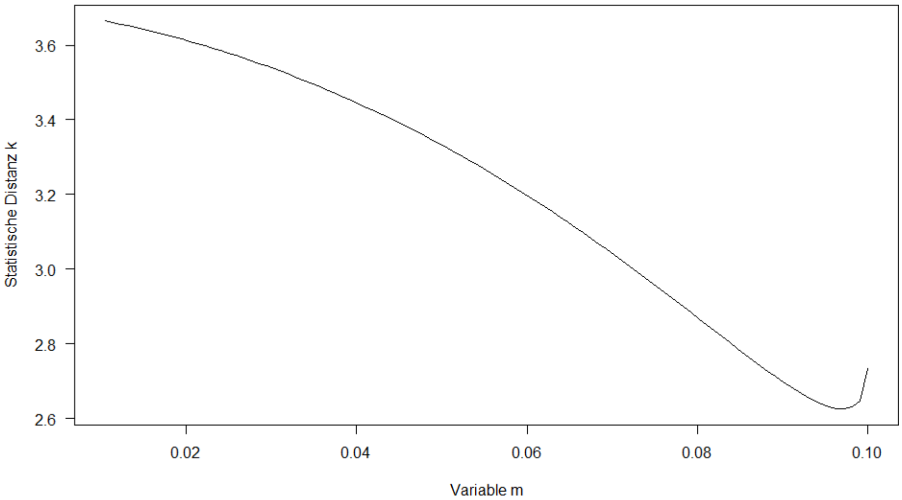


Figure 4-6: Representation of the statistical distance k as a function of the variables m

From the figure, it can be seen that the minimum statistical distance is expected to be close to the value 2.6. Using a numerical optimization procedure we obtain:

$m = 0.097\ 089\ 5$ and $k(m) = 2.625\ 029$

Thus, to calculate the minimum capability index $_{Pok,}$ we use the minimum statistical distance:

$k_{Pok} = 2.625\ 029.$